III B.Sc Statistics

Subject: Statistical quality control

Sub code: CST52

Unit: 2

UNIT-II

CONTROL CHARTS.

Theory of control charts.

INTRODUCTION:

Control chart was developed by W.A. Schewhart. A control chart is a statistical device principally used for the studying and the control of repeatative process. The control chart may serve.

1. First to define the goal (or) standard for a process. That the management might strive to attain.

2. Second it may be used as a instruement for attaining that goal.

3. Third, it may be solve as means of judging whether the goal has been reaches.

Thus it is an instruement to be used in specification, production & inspection.

Causes of variation in quality.

Quality of a product is affected by various reasons therefore variations that exist are of two kinds.

1. Variation due to Assignable Causes.

2. Variation due to Chance Causes.

Variation due to Men, Machine, Material that can be identified and rectified by human effects efforts are called variation due to assignable causes.

variation due to some inherent.

characteristics of the production and Process. That cannot be detected by human efforts is said to be variation due to chance causes.

CONTROL CHART.

DEFNITION:

A control chart is a graphical representation of the collected information. the inform may pertain to some measureable characteristic (or) Judges quality characteristics of sample It detects the variation in processing and warns of there is any departure from the specified tolerance limits.

Control chart is a device which specifies the state of statistical control. A device for attaining the statistical control. A device to judge whether the control has been attained. The control limits on the control chart are so place as to disclose the presence (or absence of the assingable causes of Quality.

They make the possible diagonosis and the correction of many products trouble switch enable the reduction of spoilage and rework.

There are many types of control charts designed for different control situations. The most commonly used control charts are,

i) control charts for measureable Quality characteristics ($\bar{x}$, R, σ)

ii) control charts for fraction of defectives (p chart).

iii) control charts for number of defects for unit (C -chart)

Objectives of control charts.

Control charts which are based on statistics techniques are used for the following purposes

1. $\bar{x}$ and R, $\bar{x}$ and σ charts are used in combination for process control $\bar{x}$ charts shows variation in the average of samples.

$\bar{x}$ chart shows the uniformly (or) consistency of the process

R σ charts shows this variation in the process

2. control charts are used to determine whether the given process is incontrol (or) if is at what dispasion

3. They are used to secure information to be used in establishing (or) changing production Procedures. Such changes may be either eliminating the assignable causes of variations.

4. They are used to secure inform when it is necessary to widen that tolerance. Sometimes the control chart shows much variability that some product is sure to be made outsided tolerances. A review situation may show that the tolerances are tighter that what. What is necessary then the appropriate action will be taken to change the specification.

5) To secure information to be used in establishing (or) changing inspection procedure (or) acceptance procedure (or) both.

6) To provide basis for current decision (or) acceptance (or) rejection of the manufacturing product.

7. Control charts help to reduce the inspection cost.

Control charts for variables.

Control charts based upon the measurements (or) quality characteristics are called as control charts for variables. Control charts for variables are often found to be a more economic means of controling quality than control charts for attributes.

The variables control charts are $\bar{x}$, R and σ charts.

Some Relationship.

Let 'μ' denotes population mean.

'σ' denotes population SD

$\bar{x}$ denotes sample mean

$\hat{\sigma}$ denotes sample SD

'R' denotes sample range.

Let $\bar{x}_1, \bar{x}_2 \ldots \bar{x}_n$ to be the means of 'n' sub groups.

$$\bar{\bar{X}} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_n}{n}$$

then,

$$\bar{\bar{x}} = \mu$$

Let $\hat{\sigma}$ be the sample SD and $\bar{\sigma}$ denotes the mean of sample SD. The relationship between $\bar{\sigma}$ & σ is given by

$C_2 = \frac{\bar{\sigma}}{\sigma}$ for practical purposes this C_2 values are computed for different sample size and tabulated

According to statistical theory sample range is related with the processed standard deviation which is given by $W = \frac{R}{\sigma}$

W is called the relative range. The Parameters are the distribution of function of the sample size 'n'.

The mean of W "d_2" and the SD is "d_3" again the mean of sample range $\bar{R}$ and the σ have the relationship.

$$d_2 = \frac{\bar{R}}{\sigma}$$

$$\Rightarrow \sigma = \frac{\bar{R}}{d_2}$$

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choice of the variable.

The variable choosen for $\bar{x}$ and R control chart should it can be measured and it can expressed in numbers. such as length in cms and weight in kgs, thickness in mm etc. In any organisation to introduce the control chart the right variables which are likely to reduce the manufacturing cost. (ie), a quality characteristic which is responsible for high rejection (or) rework are to be selected.

Basis of sub-Grouping.

The information given by the control chart depends on the basis of selection of sub groups Therefore determination of the sub-group is very important inselling of a control chart. The following factors should be considered while selecting a sub group.

1. Each sub group should be as homogeneous as possible.

2. There should be maximum oppourtunity for variation from one group to another group.

3. Samples should not be taken at exactly equal intervals of time

Size and frequency of sub-Groups.

Size of the sub-Group.

To provide the maximum homogenity with in the sub-group. The size of the subgroup should be as small as possible however, four (or) five is the most commonly accepted sub-group size. On statistical grounds even for the small ~~small~~ sub-groups the sample mean $\bar{x}$ is nearly normal when the samples are taken from the normal population. Sometimes when the objective is to make the control chart very sensitive even for small variation in the process average Large Samples of size 10 (or) 20 may be advantage However if the cost of measurement is too high then it may necessary to use smaller sample size

Frequency of the sub-group.

There are two possible ways of taking the sub-groups. Taking large samples at less frequency intervals (or) Taking smaller samples at more frequency intervals.

The selection will be governed by causes of taking and analysing measurement. However frequency of sub-group should be more at the initial stages and could be reduced when control chart is only to maintain the process over current production, Frequency of taking a sub-group may be expressed either in terms of the time such as once in an hour (or) as a proportion of the items produced such as five out of 160 (5/100)

CENTRAL LIMITS.

For plotting the control charts generally $\pm 3\sigma$ limits are selected and they are termed as Central limits. They present a band within which the dimension of the components are expected to fall within 3σ limits 99.7% of the samples from the given population are expected to fall and only 0.003% of samples from given population will fall outside the limits. This means that only three samples out of 1000 will fall outside the 3σ limits since 3 out of 1000 is a very small risk $\pm 3\sigma$ limits have been found to give good practical results.

Starting the control chart (Making and Recording the measurements)

The information given by the control chart is influenced by variations in Quality as well as variations in measurements. Any measuring systems will have its own inherent variation which should not increase due to assingable causes such as error in recording or error in reading.

calculation procedure.

A good number of samples of items are manufacturer are collected at random at regular intervals of time and their Quality characteristic (say diameter or thickness (or) length etc) are measures. For each sample the sample mean & sample range is calculated.

STEP-I

If $x_1, x_2, \ldots x_n$ are sample observations

then $\bar{x} = \dfrac{x_1 + x_2 + \ldots + x_n}{n}$

R = max. values - min value

STEP-2

calculation of grand average and $\bar{R}$

using the sample means ($\bar{x}$'s) and sample range (R) compute the grand average & range as.

$$\bar{\bar{X}} = \frac{\bar{x}_1 + \bar{x}_2 + \dots + \bar{x}_n}{n}$$

$$\bar{R} = \frac{R_1 + R_2 + \dots R_n}{n}$$

where n = number of samples taken.

STEP-3

construction of control limits

control limits for $\bar{x}$ chart:

Generally the $\pm 3\sigma$ limits are given by

$$\mu \pm \frac{3\sigma}{\sqrt{n}}$$

μ = population mean

σ = population SD

Since we consider the samples from this normal population the point estimator of μ is $\bar{\bar{x}}$

We develope estimator for σ as follows.

According to statistical theory the sample mean $\bar{x}$ and mean of sample range $\bar{R}$ and the mean of sample SD are related as

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follow

$$c_2 = \frac{\bar{\sigma}}{\sigma} \rightarrow ①$$

$$d_2 = \frac{\bar{R}}{\sigma} \rightarrow ②$$

$$\sigma = \frac{\bar{R}}{d_2}$$

Therefore the $\pm 3\sigma$ limits for $\bar{x}$ chart is

$$\bar{x} \pm \frac{3\sigma}{\sqrt{n}}$$

$$\Rightarrow \bar{x} \pm \frac{3}{\sqrt{n}} \quad \frac{\bar{R}}{d_2}$$

Let $3/\sqrt{n} \cdot 1/d_2 = A_2$

Then the $\pm 3\sigma$ limits for $\bar{x}$ chart is

$$\bar{\bar{x}} \pm A_2 \bar{R}$$

NOTE

c_2, d_2, & A_2 are taken from the standard tables.

R chart.

The sample range is related to the process SD which is given by

$$W = \frac{R}{\sigma}$$

W is a random variable and called distribution W are a functions of sample size 'n' the mean of W is d_2. Now consider

$$W = \frac{R}{\sigma}$$

where, $R = W\sigma$

The sample range R as a distribution and the SD for R is given by $\sigma_R = d_3 . \sigma$

Since σ is unknown as may estimate

$$\hat{\sigma}_R = d_3 \cdot \frac{\bar{R}}{d_2}$$

Consequently the parameters of $\bar{R}$ chart with usual 1 3σ Control limits

$$UCL = \bar{R} + 3\hat{\sigma}_R$$

$$= \bar{R} + 3d_3 \frac{\bar{R}}{d_2}$$

$$= \bar{R}\left[1 + \frac{3d_3}{d_2}\right]$$

$$UCL = \bar{R}D_4$$

Thus the L.C.L is

$$LCL = \bar{R} - 3\hat{\sigma}_R$$

$$= \bar{R} - 3d_3 \cdot \frac{\bar{R}}{d_2}$$

$$= \bar{R}\left[1 - \frac{3d_3}{d_2}\right]$$

$$LCL = \bar{R}D_3$$

Thus Draw the R Chart we consider the following lines.

$$LCL = D_3 \cdot \bar{R}$$

$$UCL = D_4 \cdot \bar{R}$$

$$CL = \bar{R}$$

NOTE:

For sample size ≤ 5, $D_3 = 0$ after drawing the control limits and control line respective graphs. If the sample mean $\bar{x}$ respective graphs. If all the points fall well with in the control limits the process is said to be under control. The points that fall above the upper control limit indicate assignable cause of variation. If the given control limit are the standard control limits thus necessary steps are to be taken to eliminate the assignable causes.

If we have to set the control limits then we leave the points that fall outside of the control limits and compute $\bar{\bar{x}}$ and $\bar{R}$ for the remaining samples using thus $\bar{\bar{x}}$ and $\bar{R}$ the how control limits are revised control limits

$\bar{\bar{x}} \pm A_2 \bar{R}$ For $\bar{x}$ chart and $\bar{R}D_4$, $\bar{R}D_3$ for $\bar{R}$ chart are constructed. Now again the plott $\bar{x}$ & R values in the respective graph and examine whether all the points fall within the control limits. If not again we revise the construction limits till all the points fall within the control limit.

Problems.

1. construct the control chart for mean & range for the following data on the basis of sample of 5 being taken every row.

samples	1	2	3	4	5	$\bar{x}$	R
1	42	65	75	78	87	69.4	45
2	42	45	68	72	90	63.4	48
3	19	24	80	81	81	57	62
4	36	54	69	79	84	64.4	48
5	42	51	57	59	78	57.4	36
6	51	74	75	78	132	82	81
7	60	60	72	95	138	85	78
8	18	20	27	42	60	33.4	42
9	15	30	39	62	84	46	69
10	69	109	113	118	153	112.4	84
11	64	90	93	109	112	93.6	48
12	61	78	94	109	136	95.6	75
13	60	74	75	78	138	85	78
14	18	25	27	40	82	38.4	64
15	42	51	57	59	154	72.6	112
16	36	52	79	98	120	77	84
17	60	61	71	96	128	83.2	68
18	15	30	39	52	74	42	59
19	20	25	27	42	53	33.4	33
20	60	60	71	90	125	81.2	65
Total						13724	1279

$$\bar{\bar{x}} = \frac{\sum \bar{x}}{n} = \frac{1372.4}{20} = 68.62$$

$$\bar{R} = \frac{\sum R}{n} = \frac{1279}{20} = 63.95$$

$\bar{x}$ – chart

$UCL = \bar{\bar{x}} + A_2 \bar{R}$

$= 68.62 + (0.5777)(63.95)$

$= 68.62 + 36.94$

$UCL = 105.56$

$L.C.L = \bar{x} - A_2 \bar{R}$

$= 68.62 - (0.5777)(63.95)$

$= 68.62 - 36.94$

$L.C.L = 31.68$

The range is $(31.68, 105.56)$

$\bar{R}$ – chart

$U.C.L = D_4 \bar{R}$

$= (2.115)(63.95)$

$U.C.L = 135.25$

$L.C.L = D_3 \bar{R}$

$= 0(63.95)$

$L.C.L = 0$

The range is $(0, 135.25)$

Revised control chart.

$$\bar{X} = \frac{1372.4 - 112.4}{19}$$

$$\bar{X} = 66.32$$

$$R = \frac{1279 - 84}{19} = \frac{1195}{19}$$

$$R = 62.89$$

$\bar{X}$ - chart.

$$U.C.L = \bar{X} + A_2 \bar{R}$$

$$= 66.32 + (0.5777)(62.89)$$

$$= 66.32 + 36.33$$

$$U.C.L = 102.65$$

$$L.C.L = \bar{X} - A_2 \bar{R}$$

$$= 66.32 - (0.5777)(62.89)$$

$$= 66.32 - 36.33$$

$$L.C.L = 29.99$$

(Range in 29.99, 102.65)

R chart

$$U.C.L = D_4 \bar{R}$$

$$= (2.115)(62.89)$$

$$U.C.L = 133.01$$

$$L.C.L = D_3 \bar{R}$$

$$= (0)(62.89)$$

$$L.C.L = 0$$

the range is (0, 133.01)

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σ - chart.

The first step in construction of a σ-chart is to average. The standard deviations of the individuals samples and take σ, the average of σ_i's and takes the $\bar{\sigma}$ as the control line on the σ-chart. The σ^2 the SD of the process is estimated as $\hat{\sigma}$, $\frac{\bar{\sigma}}{c_2}$ then adding $\frac{3\hat{\sigma}}{\sqrt{2n}}\sqrt{2(n-1)-2nc_2^2}$ is computed. The adding this value with $\bar{\sigma}$ we get the upper control limit subtracting this from $\bar{\sigma}$ we get the L.C.L

In practise we get, the UCL & LCL for σ-chart as follows

$UCL = B_4\bar{\sigma}$

$LCL = B_3\bar{\sigma}$

The values of B_3 & B_4 are taken from the standard tables.

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