

Operation Research

Subject code : CST53

Introduction to OR

OR is discipline that deals with the application advanced analytical methods to make better decisions. Further the term operational analysis is used in the British military as the part of capability development management and assurance.

(Or)

OR is an analytical method of problem solving and decision making that is useful in the management of the organisations

Origin of OR

- * In during second world war the OR model is consider
- * Operations research is involves "Research of operations" the OR is applied to

the problem that concern have
coordinate the operation with in an
organizations

* The study of man, machine
systems that have a purpose to study
for OR in the term OR involves the
application of physical Biological and
social sciences is the most quantitative
way of possible

* OR draws on the disciplines of
mathematics, Psychology and all forms
of Engineering field

definition

* operation Research is a scientific
method of providing executive department
which a quantitative base for decision
under these control

* Operation research is a scientific approach to problems solving for executive management.

* OR is the art giving to make the decisions. It use as scientific, mathematical or logical means to attempt with the problems that conform the executive with the optimum solutions to the problem

Slope of OR

- * defences operation
- * In industry
- * In agricultural
- * In traffic control
- * In hospital
- * Education and training
- * National defence services
- * Business Management

* The scope of OR is not only confined to any specific agency like defense services but today it's widely used in all industrial organisations

* It can be used to find the best solution to any problem be it is simple complex.

* Impact on economic, management Engineering and other fields

* Railways, Indian airlines, defense organisation Tata companies etc...

* Research and development

* purchasing, procurement and exploration.

* Physical distributions

Advantages of OR

- * It provides some logical and systematical approach to the problem
- * construction of models require experts from various disciplines
- * It helps in finding avenues for new research and development improvements in a system
- * through a model, the problem under consideration becomes controllable

Limitations of OR

Costly \Rightarrow As more models and resources are involved in this it has become more costly

Not Realistic \Rightarrow Everything in OR is shown as models which may not reflect in real time situation

complex \Rightarrow As more model involved in OR
It has becomes more complex to understand
and more time is consumed in OR.

Time consuming \Rightarrow As it is more complex
and more time is consumed in OR.

Linear programming

* Linear programming problem
deals with the optimization (maximization
or minimization) of a function of decision
variables.

* (variable whose values determine
the solution of a problem are called decision
variables of the problem) known as
objective function

constraints

constraints means subject to a set of
simultaneous linear equation known
as constraints

linear

The term linear means that all the variable occurring in the objective function and the constraints are the first degreee in the problems under consideration.

programming

The term programming means the process of determining the particular course of action.

(+) Mathematical formulation of LPP

If x_j ($j = 1, 2, \dots, n$) are the n decision variables of the problem and if the system is subject to m constraints,

The general mathematical model can be written in the form

$$\text{optimize } Z = f(x_1, x_2, \dots, x_n)$$

subject to $g_i(x_1, x_2, x_3, \dots, x_n) \leq, = - \geq b_i$,

$$(i = 1, 2, \dots, m)$$

$$x_1, x_2, \dots, x_n \geq 0$$

called the non negative restriction

or constraints

Problem 1

A firm produces 3 products. This product are produced on three different machines. the time required to manufacture one unit of each of the three product and the daily capacity of the three machine are given in the table below. It is required to determine the number to the manufactured for each product daily.

The profit per unit for product 1, 2 and 3 is rupees Rs 4 and Rs 3 and Rs 6 respectively. It is assumed that all the amount produced are consumed in the market. formulate the mathematical

Model for the problem.

Machine	Time per unit minutes			Machine capacity minutes/day
	Product 1	2	3	
M ₁	2	3	2	440
M ₂	4	-	3	470
M ₃	2	5	-	430

Solution

Let x_1 , x_2 and x_3 be the number units of products 1, 2 and 3 respectively

to produced this amount of product 1, 2 and 3 it required

$$2x_1 + 3x_2 + 2x_3 \text{ (Minutes of } M_1)$$

$$4x_1 + x_2 + 3x_3 \text{ (Minutes of } M_2)$$

$$2x_1 + 5x_2 + x_3 \text{ (Minutes of } M_3)$$

But the capacity of the machines

M_1 , M_2 and M_3 are 440, 470 and 430

∴ the consider are

$$2x_1 + 3x_2 + 2x_3 \leq 440$$

$$\text{Subj to } 4x_1 + 3x_3 \leq 470$$

$$2x_1 + 5x_2 \leq 430$$

and $x_1, x_2, x_3 \geq 0$ constraint

Since

The profit per unit for product

1, 2 and 3 is Rs 4, Rs 3 and Rs 6

\therefore The Total profit is $4x_1 + 3x_2 + 6x_3$

The objective is maximize the profit

The objective function is maximize

$$Z = 4x_1 + 3x_2 + 6x_3$$

\therefore The complete formulation of the
LPP is ^{optimum} Maximize $Z = 4x_1 + 3x_2 + 6x_3$

subject to the constraints

$$2x_1 + 3x_2 + 2x_3 \leq 440$$

$$4x_1 + 3x_3 \leq 470$$

$$2x_1 + 5x_2 \leq 430$$

constraints

$$x_1, x_2, x_3 \geq 0$$

Problem 2

A person wants to design the diet which will full fill his daily requirements proteins, fats and carbohydrate at the minimum cost. The choice is to be made from four different types of food. The yield per unit of this foods are given in the following table.

Food Type	yield / unit			cost / unit Rs
	Pro	Fats	Carbo	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
minimum requirement	800	200	700	

Formulate the LPP Model for the problem.

Let x_1, x_2, x_3 , and x_4 be the units of food type 1, 2, 3 and 4.

To obtain this units of food of type 1, 2, 3 and 4.

$$3x_1 + 4x_2 + 8x_3 + 6x_4 \quad \text{protein/day}$$

$$2x_1 + 2x_2 + 7x_3 + 5x_4 \quad \text{fats/day}$$

$$6x_1 + 4x_2 + 7x_3 + 4x_4 \quad \text{carbohydrate/day}$$

Since the minimum requirement of this protein, fat and carbohydrate 800, 200 and 700 respectively

$$\text{Min} = Z$$

$$3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800$$

$$\text{Max} = L$$

$$2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$$

$$6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Since the cost of the food type 1, 2, 3 and 4

are Rs 45, Rs 40, Rs 85 and Rs 65

The Total cost

$$45x_1 + 40x_2 + 85x_3 + 65x_4$$

∴ The objective function

$$\min Z = 45x_1 + 40x_2 + 85x_3 + 65x_4$$

∴ The complete formulation are the LPP

$$\text{minimum } Z = 45x_1 + 40x_2 + 85x_3 + 65x_4$$

subject to constrain

$$3x_1 + 4x_2 + 8x_3 + 6x_4 \geq 800$$

$$2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$$

$$6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$$

$$x_1, x_2, x_3, x_4 \geq 0$$

-
2. A firm produce an alloy having the following specifications

i) specific gravity ≤ 0.98

ii) chromium $\geq 8\%$

iii) melting point $\geq 450^\circ\text{C}$

The raw materials A, B and C having in the properties shown in the table

Property	Raw material		
	A	B	C
Specific gravity	0.92	0.97	1.04
Chromium melting	7.1.	13.1.	16.1.
	440°C	490°C	480°C

cost of the various raw material per unit for R6 90 for A , 280 for B
 40 for C find the proportions in which A, B and C used to obtain an alloy of desired properties with the cost of raw material is minimum

Let x_1 , x_2 and x_3 to be the property 1, 2 and 3

From this units of property

1, 2 and 3

$$0.92x_1 + 0.97x_2 + 1.04x_3 \quad \text{specific gravity}$$

$$7x_1 + 13x_2 + 16x_3 \quad \text{chromium}$$

$$440x_1 + 490x_2 + 480x_3 \quad \text{melting}$$

Since the minimum requirement of the A, B and C, 90, 280 and 40 respectively

$$0.92x_1 + 0.97x_2 + 1.04x_3 \leq 0.98$$

$$7x_1 + 13x_2 + 16x_3 \geq 8$$

$$440x_1 + 490x_2 + 480x_3 \geq 450$$

$$x_1, x_2, \text{ and } x_3 \geq 0$$

Since the cost of the property 1, 2 and 3 are Rs 90, Rs 280 and Rs 40

$$90x_1 + 280x_2 + 40x_3$$

The objective function

$$90x_1 + 280x_2 + 40x_3$$

∴ The complete formulation of the LPP

$$\min Z = 90x_1 + 280x_2 + 40x_3$$

subject to constraint

$$0.92x_1 + 0.94x_2 + 1.04x_3 \leq 0.98$$

$$7x_1 + 13x_2 + 16x_3 \geq 8$$

$$140x_1 + 190x_2 + 480 \geq 450$$

and

$$x_1, x_2, x_3 \geq 0$$

problem 4

Old hen can be bought at Rs 2
each and ^{young} hen once at Rs 5 each. The hens
lay ~~3~~ ^{young} 3 eggs per week and the
young once lay 5 eggs per week each egg
being worth 30 paise. A hen cost Rs 1
per week to feed a person has only Rs 80
to spend for hen's, how many of each
kind should he buy to ^{get} a profit of
more than Rs 6 per week. assuming that
the he cannot house more than 20 hens
formulate this as a LPP

solution

Let x_1 be old hens

x_2 be young hens

Since

that person has only Rs 80 to spend
for hens old hens Rs 2 and young hens Rs 5

$$2x_1 + 5x_2 \leq 80$$

Also since we cannot house more
than 20 hens

$$x_1 + x_2 \leq 20$$

\therefore The total scale of eggs will be

$$= 0.3(3x_1 + 5x_2)$$

Expenditure on feeding will be

$$= 1(x_1 + x_2)$$

\therefore The net profit is

$$\text{Rs}(0.3(3x_1 + 5x_2)) - (1(x_1 + x_2))$$

$$= 0.9x_1 - x_1 + 1.5x_2 - x_2$$

$$= -0.1x_1 + 0.5x_2$$

$$= (0.5x_2 - 0.1x_1) \geq 6$$

$$\therefore (0.5x_2 - 0.1x_1) \geq 6$$

\therefore the constraints

$$2x_1 + 5x_2 \leq 80$$

$$x_1 + x_2 \leq 20$$

$$0.5x_2 - 0.1x_1 \geq 6$$

\therefore The complete formulation for the problem

$$\text{Max}^{\circ} Z = 0.5x_2 - 0.1x_1$$

subject to constraints

$$2x_1 + 5x_2 \leq 80$$

$$x_1 + x_2 \leq 20$$

$$0.5x_2 - 0.1x_1 \geq 6$$

$$x_1, x_2 \geq 0$$

problem 5

A company produces refrigerators in unit 1 and washers in unit 2. The two products are produced on soldon weekly basis. The weekly production cannot exceed 25 units in unit 1.

and 36 in unit 2, give due to constraints
 60 workers are employed. A refrigerator
 required 2 man-week of labour, while a
 water heater requires 1 man-week of labour.
 The profit available is R5600 per refrigerator
 and R5400 per heater. Formulate a
 LPP problem.

$$\text{Max } z = 60x_1 + 400x_2$$

subject to constraint

$$2x_1 + 2x_2 \leq 60$$

$$x_1 \leq 25$$

$$x_2 \leq 36$$

$$\text{and } x_1, x_2 \geq 0$$

problem:

An Electronics company produces three types of parts for automatic washing machines. It purchases casting of the parts from a local foundry and then finishes the part of drilling, shaping and polishing machines.

Graphical method of the solution of a LPP

A LPP involving only two variables can be effectively solved by a graphical method or geometrical method the problem and its solutions which gives the basic concepts used in solving general LPP which many involved any finite number of variables.

The multiple solution unbounded solution infeasible solution get the solution in graphical analysis

problem:

Solve the following LPP by the graphical method

$$\text{Maximum } Z = 2x_1 + 3x_2$$

$$\text{subject to } x_1 + x_2 \leq 30$$

$$x_2 \geq 3$$

$$x_2 \leq 12$$

$$x_1 - x_2 \geq 0$$

$$0 \leq x_1 \leq 20$$

SOL

Given that $\max z = 2x_1 + 3x_2$

First consider the inequality as constraint
as equality

$$x_1 + x_2 = 30 \rightarrow ①$$

$$x_2 = 3 \rightarrow ②$$

$$x_2 = 12 \rightarrow ③$$

$$x_1 - x_2 = 0 \rightarrow ④$$

When $x_1 = 0$ sub equ ①

$$\therefore x_2 = 30$$

When $x_2 = 0$ sub equ ②

$$\therefore x_1 = 30$$

\therefore The points are $(0, 30)$ and $(30, 0)$

\therefore We have $x_2 = 3$ $x_2 = 12$

$$x_2 = 12$$

$$x_1 - x_2 = 0 \Rightarrow x_1 = x_2$$

\therefore The coordinates of B are $\sqrt{3}$

of the region ABCDF are A(3, 3), B(12, 2)

C(18, 12), D(20, 10), and E(20, 3)

The optimum tables given below
 we can find the solution for the variable
 ∴ The optimum table is

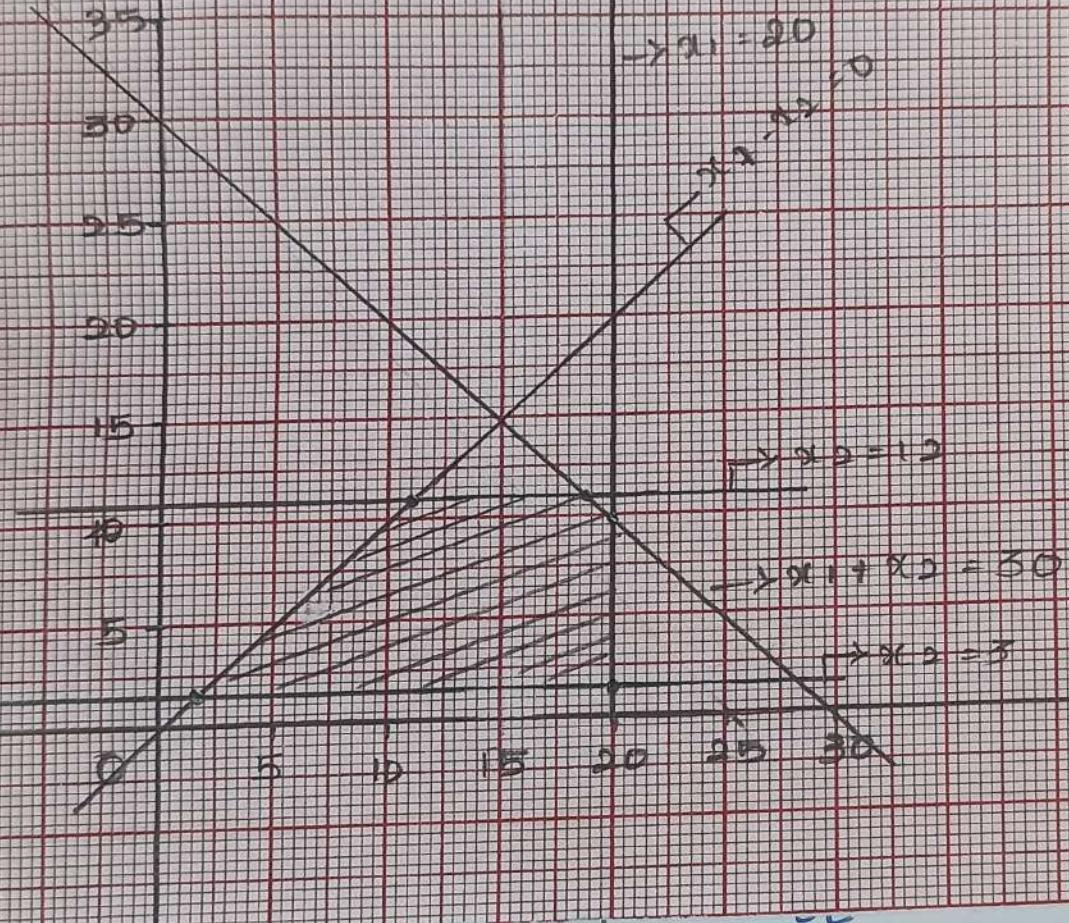
points	$Z = 2x_1 + 3x_2$	solution
A(3,3)	$Z = 2(3) + 3(3)$ $= 6 + 9$ $= 15$	$Z = 15$
B(12,12)	$Z = 2(12) + 3(12)$ $= 24 + 36$ $= 60$	$Z = 60$
C(18,12)	$Z = 2(18) + 3(12)$ $= 36 + 36$ $= 72$	$Z = 72$
D(20,10)	$Z = 2(20) + 3(10)$ $= 40 + 30$ $= 70$	$Z = 70$
E(20,3)	$Z = 2(20) + 3(3)$ $= 40 + 9$ $= 49$	$Z = 49$

The optimum tables given below

Scale

x axis = 5 unit

y axis = 5 unit



∴ The maximum value of Z is 72 and
it occurs at $(18, 12)$

The optimum solution to the LPP
is maximum $Z = 72$, $x_1 = 18$, $x_2 = 12$

problem: 2

solve the problem for LPP graphical
method.

$$\text{maximum } Z = 3x_1 + 2x_2$$

$$\text{subject to } -2x_1 + x_2 \leq 1$$

$$x_1 \leq 2$$

$$x_1 + x_2 \leq 3$$

$$\text{and } x_1, x_2 \geq 0$$

SOL Given that $\text{max } Z = 3x_1 + 2x_2$

First consider the inequality constraint
as equality

$$-2x_1 + x_2 = 1 \rightarrow ①$$

$$x_1 = 2 \rightarrow ②$$

$$x_1 + x_2 = 3 \rightarrow ③$$

when $x_1 = 0$ sub equ ①

$$\therefore -2x_1 + x_2 = 1$$

$$x_2 = 1$$

when $x_2 = 0$ sub equ ②

$$-2x_1 + 0 = 1$$

$$x_1 = -\frac{1}{2}$$

$$x_1 = 0.5$$

The points are $(0, 1)$ and $(0.5, 0)$

We have $x_1 = 2$

$$\therefore x_1 + x_2 = 3$$

The coordinates five vertices of the origin region A B C D E are A = (2, 0) B (2, 1)
C ($\frac{2}{3}, \frac{1}{3}$) D (0, 1)

y^2

scale

x axis : 1 unit

y axis : 1 unit

$$\rightarrow \alpha + \beta = 2$$

5

4

3

2

1

0

-1

-2

$$\rightarrow \alpha + \beta = 2$$

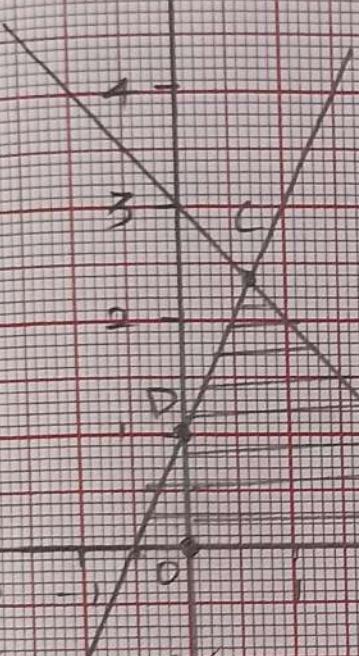
$$\rightarrow \alpha + \beta = 3$$

y

x

y

x



point	$Z = 3x_1 + 2x_2$	solution
A (2,0)	$Z = 3(2) + 2(0)$ = 6	$Z = 6$
B (2,1)	$Z = 3(2) + 2(1)$ = 6 + 2 = 8	$Z = 8$
C ($\frac{2}{3}, \frac{7}{3}$)	$Z = 3(\frac{2}{3}) + 2(\frac{7}{3})$ = $\frac{6}{3} + \frac{14}{3}$ = 6.66	$Z = 6.6$
D (0,1)	$Z = 3(0) + 2(1)$ = 2	$Z = 2$

The maximum value of Z is 8 and is

occurred at (2,1)

The optimum solution to the LPP

is maximum of $Z = 8$, $x_1 = 2$, $x_2 = 1$

PROBLEM: 3

A company makes 2 kinds of leather belt. Belt A is a high quality belt and Belt B is lower quality. The expedite property of RS4 and RS3 per belt each. But type A requires twice as much time as a belt of B and it all belts whose type B.

The company could make 1000 belt a day. The supply of leather sufficient for only 800 belts (both A and B).

Belt A requires fancy buckle and only 400 buckles per day there are only 700 buckles per day available. To determine the optimal product.

Given that Max $Z = 4x_1 + 3x_2$

First consider the inequality constraint as equality

$$2x_1 + x_2 = 1000 \rightarrow ①$$

$$x_1 + x_2 = 800 \rightarrow ②$$

$$x_1 = 400 \rightarrow ③$$

$$x_2 = 700 \rightarrow ④$$

when $x_1=0$ sub equ ①

$$2x_1 + x_2 = 1000$$

$$x_2 = 1000$$

when $x_2=0$ subequ ①

$$2x_1 + x_2 = 1000$$

$$x_1 = \frac{1000}{2}$$

$$x_1 = 500$$

the points are $(0, 1000)$ and $(500, 0)$

We have $x_1 + x_2 = 800$

$$x_1 = 400$$

$$x_2 = 400$$

The coordinates of five various ~~of the~~ the
region A, B, C D and F are A $(400, 0)$
B $(400, 200)$ C $(200, 600)$ D $(100, 700)$

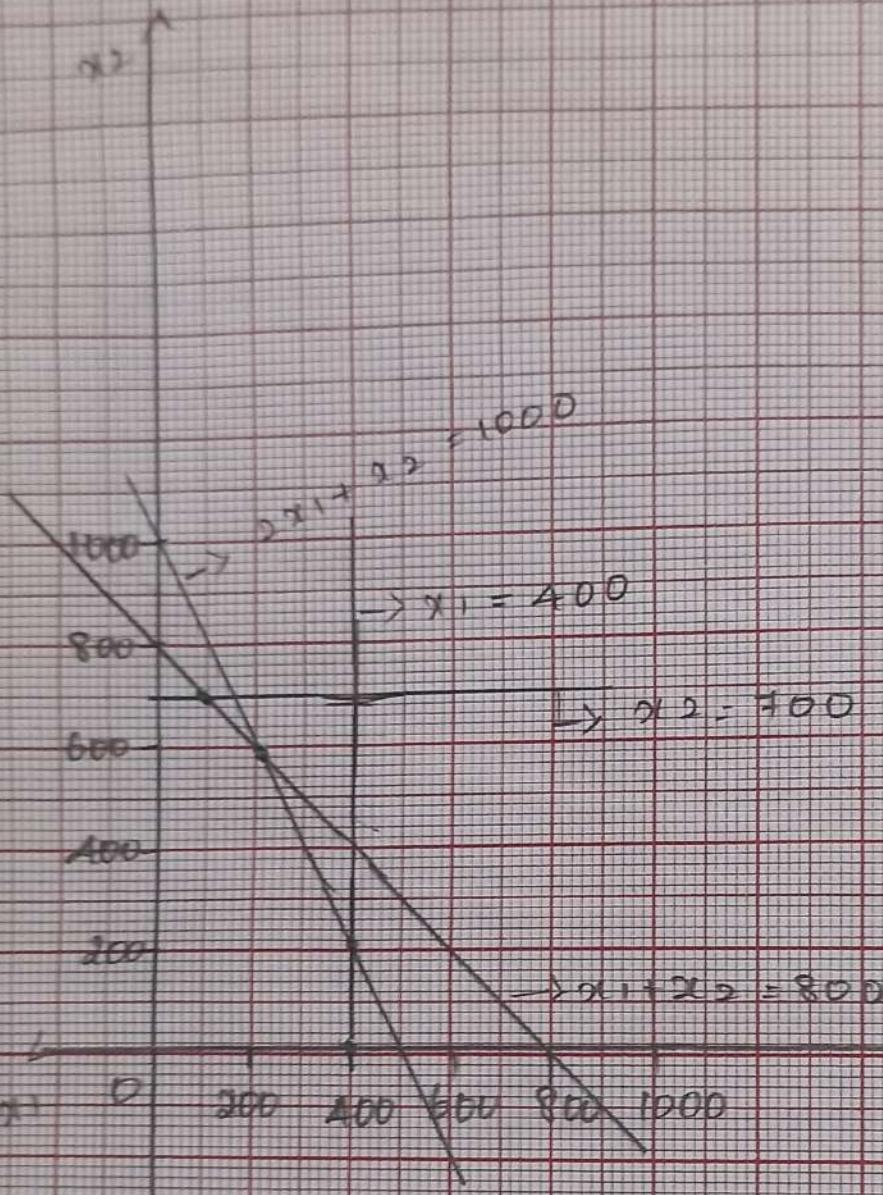
The optimum tables given below we
can find the solution for the variable

\therefore The optimum table

Point	$Z = 4x_1 + 3x_2$	solution
A(400,0)	$Z = 4x_1 + 3x_2$ $= 4(400) + 0$ $= 1600$	$Z = 1600$
B(400,200)	$Z = 4(400) + 3(200)$ $= 1600 + 600$ $= 2200$	$Z = 2200$
C(200,600)	$Z = 4(200) + 3(600)$ $= 800 + 1800$ $= 2600$	$Z = 2600$
D(100,700)	$Z = 4(100) + 3(700)$ $= 400 + 2100$ $= 2500$	$Z = 2500$

The maximum value of Z is 2600 and is obtained at (200,600)

The optimum solution of the LPP is maximum if $Z = 2600$, $x_1 = 200$, $x_2 = 600$



scale

x axis = 200 units

y axis = 200 units

4 A farm is engaged breeding pigs the
pig eat feed on various product available
on the farm. In view of the need to ensure
sufficient nutrient consist (x , y and z),
it is necessary to buy two additional
product say A and B 1 unit of product
 A contains 36 units of x , 3 units of y
and 2 units of z . 1 unit of product B
contains 6 units of x , 12 units of y and
10 units of z , the minimum requirement
of x , y and z is 108 units, 36 units
and 100 units respectively product A cost
Rs 20 per unit and product B Rs 40 per unit

Formulate the above as a LPP
problem to minimize the total cost
and solve the problem using graphic
method

SOL

Nutrient	product		minimum amount and nutrient
	A	B	
x	36	6	108
y	03	12	36
z	20	10	100

The appropriate Mathematical formulation of the LPP problem is

$$\text{Min } z = 20x_1 + 40x_2$$

subject to the constraint

$$36x_1 + 6x_2 \geq 108$$

$$03x_1 + 12x_2 \geq 36$$

$$20x_1 + 10x_2 \geq 100$$

and

$$x_1, x_2 \geq 0$$

Whole x_1 = number of units of product A
and x_2 = number of units of product B

$$36x_1 + 6x_2 = 108 \rightarrow ①$$

$$03x_1 + 12x_2 = 36 \rightarrow ②$$

$$20x_1 + 10x_2 = 100 \rightarrow ③$$

$$\frac{x_1}{3} + \frac{x_2}{18} = 1 \rightarrow ①$$

$$\frac{x_1}{12} + \frac{x_2}{3} = 1 \rightarrow ②$$

$$\frac{x_1}{5} + \frac{x_2}{10} = 1 \rightarrow ③$$

Put $x_1 = 0$ in equation ①

$$\therefore x_2 = 18$$

Put $x_2 = 0$ in equation ①

$$\therefore x_1 = 3$$

Put $x_1 = 0$ in equation ②

$$\therefore x_2 = 3$$

Put $x_2 = 0$ in equation ②

$$\therefore x_1 = 12$$