# MARUDHAR KESARI JAIN COLLEGE FOR WOMEN, VANIYAMBADI

## PG & RESEARCH DEPARTMENT OF MATHEMATICS

**CLASS : I BCA**

**SUBJECT CODE : STATISTICAL METHODS AND IT APPLICATIONS I**

**SUBJECT NAME : 23UECA12A**

## SYLLABUS

## UNIT- III

Measures of dispersion: Range, Quartile deviation, mean deviation, Standard deviation, combined Standard deviation and their relative measures.

UNIT-03

# MEASURES OF DISPERSION

Definition:

Dispersion is the measures of the variation of the item or the measurement of the scatterness of the mass of figures in a series about an average is called measured of variation or dispersion.

properties of a good measures of variation characteristics:

It should be simple to understand and easy to commute.

It should be rigidly defined

It should be based on all observations

It should be amendable to further algebraic treatment.

It must have compiling stability

It should not be affected by extreme observation

Methods of measuring Dispersion:

1. Range
2. Inter Quartilic Range
3. Mean Deviation
4. Standard Deviation
5. Lorentz Deviation

1. Range

The range is the simplest measure of Dispersion. It is a rough measure. It's measure depends upon the extreme item and not on all the item.

Range = Largest Value - Smallest Value

(i.e) $R = L - S$

Coefficient of range $= \dfrac{L-S}{L+S}$

1. find the range, coefficient of weights of 7 students from the following.

27, 30, 35, 36, 38, 40, 43

$L = 43$ Range $= L - S$

$S = 27$ $= 43 - 27 = 16$

Coefficient of Range $= \frac{L-S}{L+S} = \frac{43-27}{43+27}$

$= \frac{16}{70} = 0.2285$

Quartile Deviation

Quartile Deviation is an absolute measure of dispersion the relative measure of Dispersion known as coefficient Quartile Deviation

Coefficient of QD $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

Quartile Deviation $= \frac{Q_3 - Q_1}{2}$

Calculate Quartile Deviation and its Coefficient of monthly earnings

(X) 5m

| monthly | earnings |
|---|---|
| 1 | 239 |
| 2 | 250 |
| 3 | 251 |
| 4 | 251 |
| 5 | 257 |
| 6 | 258 |
| 7 | 260 |
| 8 | 261 |
| 9 | 262 |
| 10 | 262 |
| 11 | 673 |
| 12 | 275 |

$$\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}$$

$$\text{Coeff. Q.D} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$Q_1 = \text{Size of } \left(\frac{N+1}{4}\right)^{th} \text{ item}$

$Q_3 = \text{Size of } \left(3\left(\frac{N+1}{4}\right)\right)^{th} \text{ item}$

$N = 12$

$Q_1 = \text{Size of } \left(\frac{12+1}{4}\right)^{th} \text{ item}$

$= \text{Size of } \left(\frac{13}{4}\right)^{th} \text{ item}$

$= \text{Size of } 3.2^{th} \text{ item}$

$Q_1 = 251$

$Q_3 = \text{Size of } \left(\frac{3(12+1)}{4}\right)^{th}$

$= \text{Size of } \left(\frac{3(13)}{4}\right)^{th} \text{ item}$

$= \text{Size of } (9.75)^{th} \text{ item}$

$Q_3 = 262$

$$Q.D = \frac{Q_3 - Q_1}{2} = \frac{262 - 251}{2} = \frac{11}{2} = 5.5$$

$$\text{Co-efficient of } Q.D = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$= \frac{262 - 251}{262 + 251} = \frac{11}{513}$$

$$= 0.0214$$

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3. Mean Deviation [or] Average Deviation

Definition

Mean Deviation is the average amount of scatter of the items in a distribution from the either the mean or the median ignoring the signs of the deviation. The average is taken of the scatter is a ~~Arit~~ AM which account for the facts that measure is called mean deviation

$\therefore$ Mean Deviation $= \frac{\Sigma|D|}{N}$

Coefficient of Mean Deviation

The relation M.D or Coefficient of M.D is obtained by dividing the mean deviation by the average used for calculating MD

$$\text{coefficient of MD} = \frac{M.D}{\text{mean (or) median (or) mode}}$$

Mean Deviation (Individual)

Formula:

$$M.D = \frac{\Sigma|D|}{N}$$

where

$|D| = x - \bar{x}$

N = number of item

$\bar{x}$ = Mean

Calculate the semi inter Quartile range & Quartile Coefficient from the following

| Age in years | No of members |
|---|---|
| 20 | 3 |
| 30 | 61 |
| 40 | 132 |
| 50 | 153 |
| 60 | 140 |
| 70 | 51 |
| 80 | 3 |

| Cf |
|---|
| 3 |
| 64 |
| 196 |
| 349 |
| 489 |
| 540 |
| 543 → N value |

$$QR = \frac{Q3 - Q1}{2}$$

$$QC = \frac{Q3 - Q1}{Q3 + Q1}$$

$$Q3 = \text{Size of } \left(\frac{3(N+1)}{4}\right)^{th} \text{ item}$$

$$Q3 = \text{Size of } \left(\frac{3(544)}{4}\right)^{th} \text{ item}$$

$$= \text{Size of } (408)^{th} \text{ item}$$

$$Q3 = 50$$

$$Q1 = \text{Size of } \left(\frac{N+1}{4}\right)^{th} \text{ item}$$

$$Q1 = \text{Size of } \left[\frac{544}{4}\right]^{th} \text{ item}$$

$$Q1 = \text{Size of } (136)^{th} \text{ item}$$

$$Q1 = 40$$

$$QR = \frac{Q3 - Q1}{2} = \frac{50 - 40}{2} = \frac{10}{2} = 5$$

$$QC = \frac{Q3 - Q1}{Q3 + Q1} = \frac{50 - 40}{50 + 40} = \frac{10}{90} = 0.111$$

(7) 10m

Calculate the range QD and its coefficient

| wages | 30-32 | 32-34 | 34-36 | 36-38 | 38-40 | 40-42 | 42-44 |
|---|---|---|---|---|---|---|---|
| labour | 12 | 18 | 16 | 14 | 12 | 8 | 6 |

| x | f | cf |
|---|---|---|
| 30-32 | 12 | 12 |
| 32-34 | 18 | 30 |
| 34-36 | 16 | 46 |
| 36-38 | 14 | 60 |
| 38-40 | 12 | 72 |
| 40-42 | 8 | 80 |
| 42-44 | 6 | 86 |

$$Q_1 = l + \frac{\left(\frac{N}{4} - cf\right)}{f} \times i$$

$\frac{N}{4} = \frac{86}{4} = 21.5$

$f = 18 \quad l = 32, \quad i = 2 \quad cf = 12$

$Q = 32 + \frac{(21.5 - 12)}{18} \times 2$

$= 32 + \left\{\frac{9.5}{18}\right\} \times 2$

$= 32 + \frac{9.5}{9}$

$= 32 + 1.05$

$\boxed{Q1 = 33.05}$

$Q3 = l + \frac{\left(\frac{3N}{4} - cf\right)}{f} \times i$

$\frac{3N}{4} = 3 \times \frac{86}{4} = 3 \times 21.5 = 64.5$

$Q3 = 36 + \left(\frac{64.5 - 16}{14}\right) \times 2 \quad f = 14$

$cf = 16$

$= 36 + \frac{48.5}{7} \quad l = 36$

$= 36 + 6.928 \quad i = 2$

$Q3 = 42.928$

$QD = \frac{Q3 - Q1}{2}$

$$= \frac{42.928 - 33.05}{2}$$

$$= \frac{9.878}{2}$$

$$= 4.939$$

$$Qc = \frac{Q3 - Q1}{Q3 + Q1}$$

$$= \frac{42.928 - 33.05}{42.928 + 33.05}$$

$$= \frac{9.878}{75.978}$$

$$Qc = 0.1300$$

Calculate mean deviation from mean for the following data 100, 150, 200, 250, 360, 490, 500, 600, 671., Also Calculate Coefficient of mean deviation

$$MD = \frac{\Sigma |D|}{N}$$

$$CMD = \frac{MD}{M}$$

$$\bar{X} = \frac{100 + 150 + 200 + 250 + 360 + 490 + 500 + 600 + 671}{9}$$

$$\bar{X} = 369$$

| $X$ | $\lvert D \rvert = X - \bar{X}$ |
|---|---|
| 100 | 269 |
| 150 | 219 |
| 200 | 169 |
| 250 | 119 |
| 360 | 9 |
| 490 | 121 |
| 500 | 131 |
| 600 | 231 |
| 671 | 302 |

$X - 369$

$100 - 369 = 269$

$\Sigma D = 1570$

$$MD = \frac{\Sigma \lvert D \rvert}{N}$$

$\lvert D \rvert = X - \bar{X}$

$$\bar{X} = \frac{\Sigma X}{N}$$

$$CMD = \frac{MD}{M \to \bar{X}}$$

$$MD = \frac{\Sigma |D|}{N}$$

$$= \frac{15.70}{9}$$

$$MD = 174.44$$

$$CMD = \frac{MD}{Mean\ \bar{X}}$$

$$= \frac{174.44}{369}$$

$$CMD = 0.4727$$

calculate mean deviation from median for the following data. also calculate coefficient of mean deviation.

100, 150, 200, 250, 360, 490, 500, 600, 671

$$MD = \frac{\Sigma |D|}{N}$$

$$CMD = \frac{MD}{Mean}$$

$D = X - median$

$$median = \text{Size of } \left(\frac{N+1}{2}\right)^{th} \text{ item}$$

$$= \text{Size of } \left(\frac{9+1}{2}\right)^{th} \text{ item}$$

$$= \text{Size of } \left(\frac{\cancel{10}^{5}}{\cancel{2}_{1}}\right)^{th} \text{ item}$$

$$= \text{Size of } (5)^{th} \text{ item}$$

$$\bar{X} = 360$$

| X | $|D| = X - \bar{X}$ |
|---|---|
| 100 | 260 |
| 150 | 210 |
| 200 | 160 |
| 250 | 110 |
| 360 | 0 |
| 490 | + 130 |
| 500 | 140 |
| 600 | 240 |
| 611 | 311 |
| | $\Sigma |D| = 1,561$ |

$\bar{X} = 360$

$$MD = \frac{\Sigma |D|}{N}$$

$$= \frac{1561}{9}$$

$$MD = 173.44$$

$$CMD = \frac{MD}{Mean = \bar{x}}$$

$$= \frac{173.44}{360}$$

$$CMD = 0.48177$$

Discrete

$$M.D = \frac{\Sigma f |D|}{N}$$

$$D = x - \bar{x}$$

$$\bar{x} = \frac{\Sigma fx}{N}$$

Calculate MD from the following Data

| $x$ | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| frequency | 1 | 4 | 6 | 4 | 1 |

| $x$ | $f$ | $fx$ |
|---|---|---|
| 2 | 1 | 2 |
| 4 | 4 | 16 |
| 6 | 6 | 36 |
| 8 | 4 | 32 |
| 10 | 1 | 10 |
| | $\Sigma f = 16$ | $\Sigma f x_i = 96$ |

$$\bar{x} = \frac{\Sigma f x}{N}$$

$$= \frac{96}{16}$$

$$\bar{x} = 6$$

| $x$ | $f$ | $\bar{x}$ | $\lvert D \rvert = \lvert x - \bar{x} \rvert$ | $fD$ |
|---|---|---|---|---|
| 2 | 1 | 6 | 4 | 4 |
| 4 | 4 | 6 | 2 | 8 |
| 6 | 6 | 6 | 0 | 0 |
| 8 | 4 | 6 | 2 | 8 |
| 10 | 1 | 6 | 4 | 4 |

$\Sigma fD = 24$

$$M.D = \frac{\Sigma fD}{N}$$

$$= \frac{24}{16}$$

$$= 1.5$$

$$CMD = \frac{MD}{\bar{x}}$$

$$= \frac{1.5}{6}$$

$$= 0.25$$

Continous

$$MD = \frac{\Sigma f|D|}{N}$$

$$\bar{x} = \frac{\Sigma fm}{N}$$

Calculate MD from the following data

| class | 2-4 | 4-6 | 6-8 | 8-10 |
|---|---|---|---|---|
| frequency | 3 | 4 | 2 | 1 |

| X | m | f | fm |
|---|---|---|---|
| 2-4 | 3 | 3 | 9 |
| 4-6 | 5 | 4 | 20 |
| 6-8 | 7 | 2 | 14 |
| 8-10 | 9 | 1 | 9 |

$\Sigma f = 10$ $\Sigma fm = 52$

$$\bar{X} = \frac{\Sigma fm}{\Sigma f}$$

$$= \frac{52}{10}$$

$$= 5.2$$

| X | m | f | $\bar{X}$ | $D = \overset{(m)}{X} - \bar{X}$ | $f\,\lvert D\rvert$ |
|---|---|---|---|---|---|
| 2-4 | 3 | 3 | 5.2 | 2.2 | 6.6 |
| 4-6 | 5 | 4 | 5.2 | 0.2 | 0.8 |
| 6-8 | 7 | 2 | 5.2 | 1.8 | 3.6 |
| 8-10 | 9 | 1 | 5.2 | 3.8 | 3.8 |

$\Sigma f\,\lvert D\rvert = 14.8$

$$MD = \frac{\Sigma \lvert D\rvert}{\Sigma f}$$

$$= \frac{14.8}{10}$$

$$= 1.48$$

Standard Deviation

Individual

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left[\frac{\Sigma x}{N}\right]^2}$$

Calculate SD from the following data

14, 22, 9, 15, 20, 17, 12, 11

| $X$ | $X^2$ |
|---|---|
| 14 | 196 |
| 22 | 484 |
| 9 | 81 |
| 15 | 225 |
| 20 | 400 |
| 17 | 289 |
| 12 | 144 |
| 11 | 121 |

$\Sigma x = 120$ $\Sigma x^2 = 1940$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left[\frac{\Sigma x}{N}\right]^2}$$

$$= \sqrt{\frac{1940}{8} - \left[\frac{(120)}{8}\right]^2}$$

$= \sqrt{\frac{[illegible]}{8} - (14)^2}$

$= \sqrt{2[illegible].5 - 1[illegible]}$

$= \sqrt{17.5}$

$= 4.183$

Discrete

$SD = \sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left[\frac{\Sigma fd}{N}\right]^2}$

Calculate SD from the following data

| Marks | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|
| No of frequency | 8 | 12 | 20 | 10 | 7 | 3 |

| $x$ | $f$ | $d = x - A$ | $d^2$ | $fd$ | $fd^2$ |
|---|---|---|---|---|---|
| 10 | 8 | −20 | 400 | −160 | 3200 |
| 20 | 12 | −10 | 100 | −120 | 1200 |
| 30 | 20 | 0 | 0 | 0 | 0 |
| 40 | 10 | 10 | 100 | 100 | 1000 |
| 50 | 7 | 20 | 400 | 140 | 2,800 |
| 60 | 3 | 30 | 900 | 90 | 2700 |
| | | | | $\Sigma fd = 50$ | $\Sigma fd^2 = 10,900$ |

$$\sigma = \sqrt{\frac{\Sigma f d^2}{N} - \left[\frac{\Sigma f d}{N}\right]^2}$$

$$\sigma = \sqrt{\frac{10{,}900}{60} - \left[\frac{50}{60}\right]^2}$$

$$= \sqrt{181.66 - (0.833)^2}$$

$$= \sqrt{181.66 - 0.693}$$

$$= \sqrt{180.967}$$

$$\sigma = 13.45$$

CONTINUOUS :

$$\sigma = \sqrt{\frac{\Sigma f d^2}{N} - \left[\frac{\Sigma f d}{N}\right]^2} \times C$$

$$d' = \frac{M - A}{C}$$ C → class interval

Compute the S.D from the following data

| x | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
|---|---|---|---|---|---|---|---|
| quency | 8 | 12 | 17 | 14 | 9 | 7 | 4 |

| X | f | m | d' | $d'^2$ | fd' | $fd'^2$ |
|---|---|---|---|---|---|---|
| 0-10 | 8 | 5 | -3 | 9 | -24 | 72 |
| 10-20 | 12 | 15 | -2 | 4 | -24 | 48 |
| 20-30 | 17 | 25 | -1 | 1 | -17 | 17 |
| 30-40 | 14 | 35 | 0 | 0 | 0 | 0 |
| 40-50 | 9 | 45 | 1 | 1 | 9 | 9 |
| 50-60 | 7 | 55 | 2 | 4 | 14 | 28 |
| 60-70 | 4 | 65 | 3 | 9 | 12 | 36 |

$$\sigma = \sqrt{\frac{216}{71} - \left(\frac{30}{71}\right)^2} \times 10$$

$$= \sqrt{2.957 - (0.422)^2} \times 10$$

$$= \sqrt{2.957 - 0.178} \times 10$$

$$= \sqrt{2.779} \times 10$$

$$= \cancel{1}.6670 \times 10$$

$$= 16.67.$$

## Co-efficient of variation [Relative S.D]

**Definition**

The measure of dispersion based on S.D is defined by $\frac{S.D}{\bar{x}} \times 100$. It is called a coefficient of variation

$$C.V = \frac{S.D}{mean} \times 100 \quad [or]$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

## Combined standard deviation

A sample of $N_1$ item as a mean $\bar{x}_1$ and $\sigma_1$ and another sample of $N_2$ items as a mean $\bar{x}_2$ and $\sigma_2$.

We can find out the combined mean and combined standard deviation by using the formula;

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$\sigma_{12} = \sqrt{\frac{N_1 \sigma_1^2 + N_2 \sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}}$$

where

$\sigma_{12}$ - combined S.D.

$d_1 = \bar{x}_1$ - combined mean

$d_2 = \bar{x}_2$ - combined mean

Example: 1

For a group of 50 male workers, the mean and the S.D of their wages are Rs 63 and Rs 9 respectively.

For a group of 40 female workers those are Rs 54 and Rs 6 respectively. Find the S.D for the combined group of 90 workers.

Solution:

| characteristics | Groups | | combined group |
|---|---|---|---|
| | male | female | |
| Size | $N_1 = 50$ | $N_2 = 40$ | $N_1 + N_2 = 90$ |
| Mean | $\bar{X}_1 = 63$ | $\bar{X}_2 = 54$ | $\bar{X}_{12} = ?$ |
| S.D | $\sigma_1 = 9$ | $\sigma_2 = 6$ | $\sigma_{12} = ?$ |

We know that combined mean

$N_1 = 50$ $\quad \bar{X}_1 = 63$

$N_2 = 40$ $\quad \bar{X}_2 = 54$

$$\bar{X}_{12} = \frac{N_1 \bar{X}_1 + N_2 \bar{X}_2}{N_1 + N_2}$$

$$= \frac{50(63) + 40(54)}{50 + 40}$$

$$= \frac{3150 + 2160}{90}$$

$$= \frac{5310}{90}$$

$$\overline{X_{12}} = 59$$

we know that combined S.D

$$\sigma_1^2 = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}}$$

$d' = \bar{x}_1$ = Combined mean $[\bar{x}_{12}]$

$= 63 - 59$

$d_1 = 4$

$d_2 = \bar{x}_2$ - Combined mean $[\bar{x}_{12}]$

$= 54 - 59 = -5$

$$\sigma_{12} = \sqrt{\frac{50(9)^2 + 40(6)^2 + 50(4)^2 + 40(-5)^2}{50+40}}$$

$$= \sqrt{\frac{50(81) + 40(36) + 50(16) + 40(25)}{90}}$$

$$= \sqrt{\frac{4050 + 1440 + 800 + 1000}{90}}$$

$$= \sqrt{\frac{7290}{90}}$$

$$= \sqrt{81}$$

$$\boxed{\sigma_{12} = 9}$$