MARUDHAR KESARI JAIN COLLEGE FOR WOMEN, VANIYAMBADI

PG & RESEARCH DEPARTMENT OF MATHEMATICS

CLASS : I BCA

SUBJECT CODE : STATISTICAL METHODS AND IT APPLICATIONS I

SUBJECT NAME : 23UECA12A

SYLLABUS

UNIT- IV

Measures of Skewness: Karl Pearson's, Bowley's, and kelly's and coefficient of Skewness and kurtos is based on moments.

UNIT-4

Measures Of Skewness.

Skewness:

Skewness is a measure to study a statistical distribution. If a distribution is not symmetrical is called skew.

If the frequency ~~area~~ curve as a long tail to the right is called skew to the right.

If the frequency curve as a long tail to the left is called skew to the left.

1. Pearson's co-efficient of skewness:

$$= \frac{\text{Mean} - \text{Mode}}{\text{standard deviation}} \quad [\text{or}] \quad = \frac{3(\text{mean} - \text{median})}{\text{Standard deviation}}$$

Emprical Relation: [It is used when there is 2 same no in Analysis table]

$$\text{Mean} - \text{Mode} = 3(\cancel{\text{mode}}\ \text{mean} - \text{median})$$

2. Bowley's co-efficient Of skewness:

$$= \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1} \quad [\text{or}] \quad \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

Where ; $Q_1 = l_1 + \dfrac{N/4 - m_1}{f_1} \times c$

$$Q_2 = l_2 + \frac{N/2 - m_2}{f_2} \times c$$

$$Q_3 = l_3 + \frac{3\frac{N}{4} - m_3}{f_3} \times c$$

where. l_1 - lower limit of the Q_1 class

m_1 - c.f of preceeding class

f_1 - frequency of the Q_1 class

c - class Interval

N - Total frequency

Q_2 - Second quartile is called median.

	Individual	Discrete	Continuous.
Mean ($\bar{x}$)	$\bar{x} = \dfrac{\Sigma x}{N}$ N - no. of. observation	$\bar{x} = \dfrac{\Sigma fx}{N}$ N - total frequency.	$\bar{x} = A + \dfrac{\Sigma fd}{N} \times c$ where, A - assumed mean N - tot. frequency c - class Interval $d = \dfrac{x_i - A}{c}$
Median (M)	ODD $\dfrac{n+1}{2}$ EVEN $\dfrac{n}{2}$ and $\left(\dfrac{n}{2}+1\right)$	$\bar{x} = \dfrac{\Sigma fx}{N}$ N - total frequency	$M = l + \dfrac{\frac{N}{2} - m}{f} \times c$

	Individual	Discrete	Continuous
Mode			$\text{mode} = l + \frac{f_1 - f_0}{2f_1 - (f_0 + f_2)} \times c$ where, l - lower limit of the model class f_1 - freq. of model class f_0 - freq. of premodel class f_2 - " " postmodel class. c - class Interval
	$\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$ (or) $\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$	$\sigma = \sqrt{\frac{\Sigma f(x_i - \bar{x})^2}{N}}$ (or) $\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$	$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$ $\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$ $\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{N}\right)^2} \times c$

1. Find the pearson's co-efficient of skewness for the following frequency distribution:

Annual sale (in '000 Rs)	No. of items
0-20	20
20-40	50
40-60	59
60-80	30
80-100	25
100-120	16

X	Mid value	f	$d = \frac{x-A}{c}$	d^2	fd	fd^2
0-20	10	20	-2 $\frac{0-50}{20}$	4	-40	80
20-40	30	f_0 50	-1 $\frac{20-50}{20}$	1	-50	50
40-60	A 50	f_1 59	0 $\frac{40-50}{20}$	0	0	0
60-80	70	f_2 30	1	1	30	30
80-100	90	25	2	4	50	100
100-120	110	16	3	9	48	144
		N=200			$\sum fd =$ 38	$\sum fd^2 =$ 404

[or A = 70]

$$\text{Mean} = A + \frac{\sum fd}{N} \times c$$

$A = 50, \sum fd = 38, N = 200, c = 20$

$$= 50 + \frac{38}{200} \times 20$$

$$= 50 + 3.8$$

$$\boxed{\text{Mean} = 53.8}$$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - (f_0 + f_2)} \times c$$

$l = 40, f_1 = 59, f_2 = 30, f_0 = 50$

$$= 40 + \frac{59 - 50}{2(59) - (50 + 30)} \times 20$$

$$= 40 + \frac{9}{38} \times 20$$

$$= 40 + 4.73$$

$\boxed{\text{Mode} = 44.73}$

standard deviation (σ) = $\sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} \times c$

$= \sqrt{\frac{404}{200} - \left(\frac{38}{200}\right)^2} \times 20$

$= \sqrt{2.02 - 0.0361} \times 20$

$= \sqrt{1.9839} \times 20$

$= 1.408 \times 20$

$\boxed{\text{standard deviation} = 28.16.}$

pearson's co-efficient = $\frac{\text{Mean} - \text{Mode}}{\text{standard deviation}}$

$= \frac{53.8 - 44.73}{28.16}$

$= \frac{9.07}{28.16} = 0.32$

$\boxed{\text{pearson's coefficient} = 0.32.}$

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2. Calculate the pearson's coefficient of skewness for the following data: (Individual)

25, 15, 23, 40, 27, 25, 23, 25, 20

Sol:-

x	$d = x - A$	d^2
25	-2	4
15	-12	144
23	-4	16
40	13	169
27 A	0	0
25	-2	4
23	-4	16
25	-2	4
20	-7	49
$\Sigma f = 223$	$\Sigma d = -20$	$\Sigma d^2 = 406$

$$\text{Mean} = \frac{\Sigma x}{N}$$

$$= \frac{223}{9}$$

$$\boxed{\bar{x} = 24.7}$$

$$\boxed{\text{Mode} = 25}$$ Individual → Repeated Values

standard deviation (σ) =

$$\sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$$= \sqrt{\frac{406}{9} - \left(\frac{-20}{9}\right)^2}$$

$$= \sqrt{45.11 - (-2.22)^2}$$

$$= \sqrt{45.11 - 4.92}$$

$$= \sqrt{40.19}$$

$$\boxed{\text{Standard deviation} = 6.339.}$$

$$\text{pearson's co-efficient} = \frac{\text{Mean} - \text{Mode}}{\text{S.D}}$$

$$= \frac{24.7 - 25}{6.339}$$

$$= \frac{-0.3}{6.339}$$

$$\boxed{\text{pearson's co-efficient} = -0.04}$$

3. Calculate the pearsons co-efficient of skewness of the following data:

7, 4, 10, 9, 15, ~~14~~, 12, 7, 9, 7

Sol.:

x	$d = x - A$	d^2
7	-8	64
4	-11	121
10	-5	25
9	-6	36
15 A	0	0
12	-3	9
7	-8	64
9	-6	36
7	-8	64
$\Sigma x = 80$	$\Sigma d = -55$	$\Sigma d^2 = 419$

$$\text{Mean} = \frac{\Sigma x}{N}$$

$$= \frac{80}{9}$$

$$\boxed{\text{Mean} = 8.88}$$

$$\boxed{\text{Mode} = 7}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$$= \sqrt{\frac{419}{9} - \left(\frac{-55}{9}\right)^2}$$

$$= \sqrt{46.55 - (-6.11)^2}$$

$$= \sqrt{46.55 - 37.33}$$

$$= \sqrt{9.22}$$

$$\boxed{\text{Standard deviation} = 3.03}$$

$$\text{Pearson's co-efficient} = \frac{\text{Mean} - \text{Mode}}{\text{S.D}}$$

$$= \frac{8.88 - 7}{3.03} = \frac{1.88}{3.03}$$

$$\boxed{\text{pearson's co-efficient} = 0.62}$$

4. Find the pearson's co-efficient of skewness from the following data:

x	3	4	5	6	7	8	9	10
Size	7	10	14	35	102	136	43	8

Sol:

x	f	fx	d = x−A = x−6	d^2	fd	fd^2
3	7	21	−4	16	−28	112
4	10	40	−3	9	−30	90
5	14	70	−2	4	−28	56
6	35	210	−1	1	−35	35
A 7	102	714	0	0	0	0
8	136	1088	1	1	136	136
9	43	387	2	4	86	172
10	8	80	3	9	24	72
	$\Sigma f = 355$	$\Sigma fx = 2610$			$\Sigma fd = 125$	$\Sigma fd^2 = 673$

$$\text{Mean} = \frac{\Sigma fx}{N}$$

$$= \frac{2610}{355}$$

$$\boxed{\text{Mean} = 7.352}$$

$$\boxed{\text{Mode} = 3}$$

Mode = Initial value (3)

$$S.D = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{673}{655} - \left(\frac{125}{355}\right)^2}$$

$$= \sqrt{1.895 - 0.123}$$

$$= \sqrt{1.772}$$

$$\boxed{S.D = 1.331}$$

$$\text{pearson's co-efficient} = \frac{\text{Mean} - \text{Mode}}{\text{standard deviation}}$$

$$= \frac{7.352 - 3}{1.331}$$

$$= \frac{4.352}{1.337}$$

$$\boxed{\text{pearson's co-efficient} = 3.269}$$

5. Find the pearson's co-efficient of skewness from following data :

class	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89
frequency	5	9	14	20	25	15	8	4

Sol :-

x	m	f	$d = \frac{m-A}{c}$	d^2	fd	fd^2
9.5-19.5	14.5	5	$\frac{14.5-54.5}{10} = -4$	16	-20	80
19.5-29.5	24.5	9	-3	9	-27	81
29.5-39.5	34.5	14	-2	4	-28	56
39.5-49.5	44.5	20 f_0	-1	1	-20	20
l 49.5-59.5	54.5 A	25 f_1	0	0	0	0
59.5-69.5	64.5	15 f_2	1	1	15	15
69.5-79.5	74.5	8	2	4	16	32
79.5-89.5	84.5	4	3	9	12	36
		$\Sigma f = 100$			$\Sigma fd = -52$	$\Sigma fd^2 = 320$

$$\text{Mean} = A + \frac{\Sigma fd}{N} \times c$$

$$= 54.5 + \frac{(-52)}{100} \times 10$$

$$= 54.5 - 5.2$$

$$\text{Mean} = 49.3$$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - (f_0 + f_2)} \times c$$

$l = 49.5$; $f_1 = 25$; $f_0 = 20$; $f_2 = 15$

$$2f_1 - f_0 - f_2$$

$$= 49.5 + \frac{25 - 20}{2(25) - (20 + 15)} \times 10$$

$$2(25) - 20 - 15$$

$$50 - 20 - 15$$

$$30 - 15$$

$$15$$

$$= 49.5 + \frac{5}{\cancel{15}\,45} \times 10$$

$$49.5 + \frac{\cancel{5}\,1}{\cancel{15}\,3} \times 10$$

$$\boxed{52.83}$$

Bowley's co-efficient of skewness:

1. Find Out the Bowley's co-efficient of skewness from the following data:

* 21 - midvalue
3 ← → 3
18 - 24 (m = 21)

21 - 27 → (difference 6)

Mid value	21	27	33	39	45	51	57
frequency	18	22	40	50	38	12	4

Sol:

C.I	Mid value	f	cf
18 - 24	21	18	18
24 - 30	27	22	m_1 40
l_1 30 - 36	33	f_1 40	m_2 80 Q_1
l_2 36 - 42	39	f_2 50	m_3 130 Q_2
l_3 42 - 48	45	f_3 38	168 Q_3
48 - 54	51	12	180
54 - 60	57	4	184
		$\Sigma f = 184$	

$$B = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$Q_1 = l_1 + \frac{N/4 - m_1}{f_1} \times c$$

$$= 30 + \frac{184/4 - 40}{40} \times 6$$

$$= 30 + \frac{6}{40} \times 6$$

$$= 30 + 0.9$$

$$\boxed{Q_1 = 30.9}$$

$$Q_3 = l_3 + \frac{3\frac{N}{4} - m_3}{f_3} \times c$$

$$= 42 + \frac{3\left(\frac{184}{4}\right) - 130}{38} \times 6$$

$$= 42 + \frac{138 - 130}{38} \times 6$$

$$= 42 + \frac{8}{38} \times 6$$

$= 42 + 1.26$

$$\boxed{Q_3 = 43.26}$$

$$Q_2 = l_2 + \frac{\frac{N}{2} - m_2}{f_2} \times c$$

$$= 36 + \frac{\frac{184}{2} - 80}{50} \times 6$$

$$= 36 + \frac{12}{50} \times 6$$

$$= 36 + 1.44$$

$$\boxed{Q_2 = 37.44}$$

Bowley's co-efficient of skewness:

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{43.26 + 30.9 - 2(37.44)}{43.26 - 30.9}$$

$$= \frac{74.16 - 74.88}{12.36}$$

$$= \frac{-0.72}{12.36}$$

$$\boxed{\text{Bowley's co-effi.} = -0.058}$$

2.

Payments of commission	No. of sales man
100-120	4
120-140	10
140-160	16
160-180	29
180-200	52
200-220	80
220-240	42
240-260	43
260-280	17
280-300	7

Sol:-

C.I	f	cf
100-120	4	4
120-140	10	14
140-160	16	30
160-180	29	59 m_1
l_1 180-200	52 f_1	m_2 111 Q_1
l_2 200-220	80 f_2	m_3 191 Q_2
l_3 220-240	42 f_3	233 Q_3
240-260	43	256
260-280	17	273
280-300	7	280
	$\Sigma f = 280$.	

$$Q_1 = \left(\frac{N}{4}\right)^{th} \text{item} = \frac{280}{4} = 70$$

$$Q_2 = \left(\frac{N}{2}\right)^{th} \text{item} = \frac{280}{2} = 140$$

$$Q_3 = 3\left(\frac{N}{4}\right)^{th} \text{item} = 3(70) = 210$$

$$Q_1 = l_1 + \frac{\frac{N}{4} - m_1}{f_1} \times c$$

$$= 180 + \frac{70 - 59}{52} \times 20$$

$$= 180 + \frac{11}{52} \times 20$$

$$= 180 + 4.23$$

$$\boxed{Q_1 = 184.23}$$

$$Q_2 = l_2 + \frac{\frac{N}{2} - m_2}{f_2} \times c$$

$$= 200 + \frac{140 - 111}{80} \times 20$$

$$= 200 + 0.3625 \times 20$$

$$= 200 + 7.25$$

$$\boxed{Q_2 = 207.25}$$

$$Q_3 = l_3 + \frac{3\left(\frac{N}{4}\right) - m_3}{f_3} \times c$$

$$= 220 + \frac{210 - 191}{42} \times 20$$

$$= 220 + \frac{19}{42} \times 20$$

$$= 220 + 9.04$$

$$\boxed{Q_3 = 229.04}$$

Bowley's co-efficient of skewness

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{229.04 + 184.23 - 2(207.25)}{229.04 - 184.23}$$

$$= \frac{413.27 - 414.5}{44.81}$$

$$= \frac{-1.23}{44.81}$$

$\boxed{\text{Bowley's co-efficient} = -0.0274.}$

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3. For a distribution Bowley's coefficient of skewness is -0.36. lower Quartile is 8.6 and Median is 12.3. What is the Quartile co-efficient of dispersion.

Sol:

Quartile co-efficient of dispersion $= \frac{Q_3 - Q_1}{Q_3 + Q_1} \rightarrow (1)$

Gn:

Bowley's coefficient $= -0.36$

$Q_1 = 8.6$

$Q_2 = 12.3$

Bowley's coefficient of skewness $= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$

$$-0.36 = \frac{Q_3 + 8.6 - 2(12.3)}{Q_3 - 8.6}$$

$$-0.36(Q_3 - 8.6) = Q_3 + 8.6 - 24.6$$

$$-0.36\,Q_3 + 3.096 = Q_3 . 16$$

$$3.096 + 16 = Q_3 + 0.36 Q_3$$

$$19.096 = Q_3(1 + 0.36)$$

$$19.096 = Q_3(1.36)$$

$$Q_3 = \frac{19.096}{1.36}$$

$$\boxed{Q_3 = 14.041}$$

Sub Q_3 and Q_1 in equ (1)

$$\text{Co-efficient of dispersion} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$= \frac{14.0411 - 8.6}{14.0411 + 8.6}$$

$$= \frac{5.4411}{22.6411}$$

$$\boxed{\text{Coefficient of dispersion} = 0.2403}$$

4. Find out the Bowley's coefficient of skewness from the following data :

Mid Value	75	100	125	150	175	200	225	250
frequency	35	40	48	100	125	80	50	22

Sol:

C.I	M	f	cf
62.5 - 87.5	75	35	35
87.5 - 112.5	100	40	75
112.5 - 137.5	125	48	m_1 123
l_1 137.5 - 162.5	150	100 f_1	m_2 223 Q_1
l_2 162.5 - 187.5	175	125 f_2	m_3 348 Q_2
l_3 187.5 - 212.5	200	80 f_3	428 Q_3
212.5 - 237.5	225	50	478
237.5 - 262.5	250	22	500
		$\Sigma f = 500$	

$$Q_1 = \left(\frac{N}{4}\right)^{th} \text{item} = \frac{500}{4} = 125$$

$$Q_2 = \left(\frac{N}{2}\right)^{th} \text{item} = \frac{500}{2} = 250$$

$$Q_3 = 3\left(\frac{N}{4}\right)^{th} \text{item} = 3(125) = 375$$

$$Q_1 = l_1 + \frac{\frac{N}{4} - m_1}{f_1} \times c$$

$$= 137.5 + \frac{\frac{500}{4} - 123}{100} \times 25$$

$= 137.5 + \frac{.2}{100} \times 25$

$= 137.5 + 0.5$

$\boxed{Q_1 = 138}$

$$Q_2 = l_2 + \frac{\frac{N}{2} - m_2}{f_2} \times c$$

$$= 162.5 + \frac{250 - 223}{125} \times 25$$

$$= 162.5 + \frac{27}{125} \times 25$$

$= 162.5 + 5.4$

$\boxed{Q_2 = 167.9}$

$$Q_3 = l_3 + \frac{3\left(\frac{N}{4}\right) - m_3}{f_3} \times c$$

$$= 187.5 + \frac{375 - 348}{80} \times 25$$

$$= 187.5 + \frac{27}{80} \times 25$$

$= 187.5 + 8.437$

$\boxed{Q_3 = 195.93}$

Bowley's Co-efficient Of skewness

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{195.93 + 138 - 2(167.9)}{195.93 - 138}$$

$$= \frac{333.93 - 335.8}{57.93}$$

$$= \frac{-1.87}{57.93}$$

$$\boxed{\text{Bowley's Coefficient} = -0.032}$$

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5 In distribution mean = 65, Median = 70 and Co-efficient of skewness is -0.6. Find

i) Mode

ii) Co-efficient of Variation

Sol.

Gn:

Mean $\bar{x} = 65$

Median = 70

Coefficient Of skewness = -0.6

$$\text{Co-efficient Of Variation} = \frac{\sigma}{\bar{x}} \times 100 \longrightarrow (1)$$

$$\text{Co-efficient Of skewness} = \frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}}$$

$$\text{skewness} = \frac{3(65-70)}{S.D}$$

$$S.D = \frac{3(65-70)}{\text{skewness}}$$

$$\sigma = \frac{-15}{-0.6}$$

$$\boxed{\sigma = 25}$$

Sub $\sigma = 25$ in equ (1)

$$\text{Coefficient Of Variation} = \frac{25}{65} \times 100$$

$$= 0.3846 \times 100$$

$$= 38.46$$

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

$$65 - \text{Mode} = 3(65-70)$$

$$65 - \text{Mode} = 3(-5)$$

$$-\text{Mode} = -15 - 65$$

$$-\text{Mode} = -80$$

$$\boxed{\text{Mode} = 80}$$

6. From a cube distribution the Mean Value is Rs 20 and the median price is Rs 17 if the Co-efficient of Variation is the 20 %. Find the pearson's co-efficient of skewness.

Sol:-

$$\text{Coefficient Of Variation} = \frac{\sigma}{\bar{x}} \times 100$$

$$\text{Co-efficient Of variation} \times \frac{\bar{x}}{100} = \sigma$$

$$\sigma = \frac{20}{100} \times \frac{20}{100}$$

$$= \frac{400}{10,000}$$

$$\boxed{\sigma = 0.04}$$

$$\text{pearson's co-efficient Of skewness} = \frac{3(\text{Mean} - \text{Median})}{S.D}$$

$$= \frac{3(20-17)}{0.04}$$

$$= \frac{3(3)}{0.04}$$

$$= \frac{.9}{0.04}$$

$$\boxed{\text{pearson's co-efficient} = 225}$$

In a distribution sum of two quadrants is $\overset{Q_3+Q_1}{78.2}$ and its difference is $\overset{Q_3-Q_1}{14.3}$ and if its median is $\overset{Q_2}{35.7}$ find the coefficient of skewness.

Sol:

$$Q_3 + Q_1 = 78.2$$

$$Q_3 - Q_1 = 14.3$$

$$Q_2 = 35.7$$

$$\text{Coefficient of skewness} = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

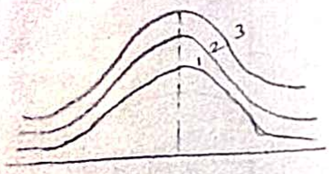
$$= \frac{78.2 - 2(35.7)}{14.3}$$

$$= \frac{78.2 - 71.4}{14.3}$$

$$\boxed{\text{Coefficient of skewness} = 0.4755}$$

Kurtosis Based On Moments:

Kurtosis is a measure of platness or peakness of a distribution



1 - M
2 - P
3 - L

Types of kurtosis:

1. Mesokurtic
2. Platykurtic
3. Leptokurtic.

1. Mesokurtic:

Normal curve (Bell shaped) is called Mesokurtic

2. Platykurtic:

The curve which is more flat topped that the normal curve is called platykurtic.

3. Leptokurtic:

The curve which is more peaked than the normal curve is called leptokurtic.

Measures Of kurtosis :-

The measures of kurtosis of a frequency distribution are based upon the fourth moment about the mean of the distribution.

i.e:

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

where,

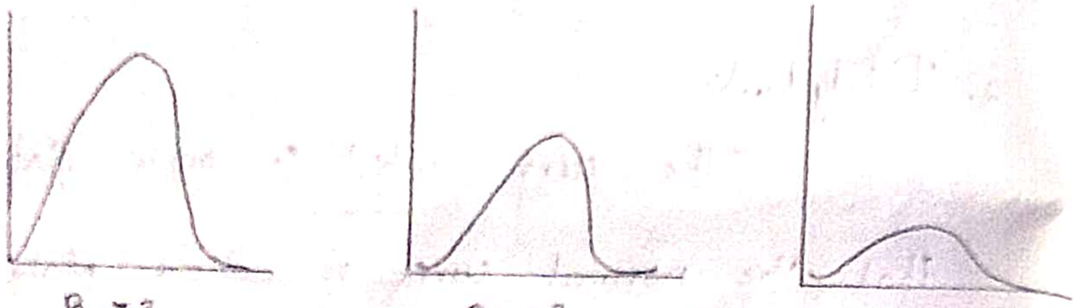
μ_4 - 4^{th} moment.

μ_2 - 2^{nd} moment.

* If $\beta_2 = 3$, the distribution is said to be normal. and the curve is a normal curve. (mesokurtic

* If $\beta_2 > 3$, the distribution is said to be more peaked. and the curve is leptokurtic.

* If $\beta < 3$, the distribution is said to be flat topped and the curve is platykurtic.



$\beta_2 > 3$ Lepto

$\beta_2 = 3$ Meso

$\beta_2 < 3$ platy

* Central moments $= \mu_r'$ $[\mu_1 = \mu_1']$

$$\beta_1 = \frac{\mu_3'^2}{\mu_2^3} \; ; \quad \mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \qquad \mu_1 = \mu_1'$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$

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1. The first 4 central moments of a distribution are 0, 2.5, 0.7, 8.75. Test the skewness and kurtosis of a distribution.

Sol:

$$\mu_1' = 0 \; ; \; \mu_2' = 2.5 \; ; \; \mu_3' = 0.7 \; ; \; \mu_4' = 8.75$$

The coefficient of skewness, $\beta_1 = \dfrac{\mu_3'^2}{\mu_2'^3}$

$$\beta_1 = \frac{(0.7)^2}{(2.5)^3}$$

$$= \frac{0.49}{15.625}$$

$$\boxed{\beta_1 = 0.031}$$

$\therefore$ β_1 is positive, the distribution is positively skewed.

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$

$$= 0.7 - 3(2.5)(0) + 2(0)^3$$

$$\mu_3 = 0.7$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$

$$= 2.5 - (0)^2$$

$$\mu_2 = 2.5$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 8.75 - 4(0.7)(0) + 6(2.5)(0)^2 - 3(0)^4$$

$$\mu_4 = 8.75$$

$$\mu_2 = (2.5)^2$$

$$\beta_2 = \frac{8.75}{(2.5)^2} = \frac{8.75}{6.25}$$

$\boxed{\beta_2 = 1.4}$

$\therefore$ The gn kurtosis is platykurtic.

---

2. The first 4 moments of the distribution about the values are 2, 20, 40, 150. find the Measures of kurtosis and comment the distribution

Sol:-

$\mu_1' = 2$, $\mu_2' = 20$, $\mu_3' = 40$, $\mu_4' = 150$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 20 - (2)^2$

$= 20 - 4$

$\mu_2 = 16$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= 40 - 3(20)(2) + 2(2)^3$

$= 40 - 120 + 16$

$\mu_3 = -64$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 150 - 4(40)(2) + 6(20)(2)^2 - 3(2)^4$

$\mu_4 = 262$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$= \frac{(-64)^2}{(16)^3}$$

$$= \frac{4096}{4096}$$

$\beta_1 = 1$ $\therefore$ β_1 is positive, the distribution is positively skewed

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{262}{(16)^2}$$

$$= \frac{262}{256}$$

$\beta_2 = 1.023$

$\therefore$ The given kurtosis is platykurtic

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2 Given $Q_1 = 18$; $Q_3 = 25$; mode $= 21$ and mean $= 18$.

Find coefficient of skewness

Sol:

Coefficient of ~~skewness~~ mode $= 3(\text{median}) - 2(\text{mean})$

$21 = 3(m) - 36$

$3 \text{ median} = 57$

Median = 19

$$\text{Coeff. of } sk = \frac{Q_3 + Q_1 - 2(Q_2)}{Q_3 - Q_1}$$

$$= \frac{25 + 18 - 2(19)}{25 - 18}$$

$$= \frac{25 + 18 - 38}{7}$$

$$= \frac{5}{7}$$

Coefficient of sk = 0.714.

---

1. From a moderately skewed distribution of retail prices for men's shoes, it is found that the mean price is Rs. 20 and the median price is Rs. 17. If the coefficient of variation is 20%, find the pearsonian sk of distribution.

Sol:

$$\text{pearson's co-efficient of } sk = \frac{3(\text{mean} - \text{median})}{S.D}$$

mean = 20 ; median = 17

$$C.V = \frac{S.D \times 100}{\text{mean}}$$

$$20 = \frac{SD \times 100}{20}$$

$$\frac{20}{100} = \frac{\sigma}{20} \times 100$$

$$\frac{1}{5} = \frac{\sigma}{20} \times 100$$

$$20 = \frac{\sigma}{20} \times 100$$

$$\frac{20}{5} = 100\sigma$$

$$5\sigma = 20$$

$$4 = 100\sigma$$

$$\sigma = \frac{20}{5}$$

$$\sigma = \frac{4}{100}$$

$$\sigma = 4$$

$$\sigma = 0.04$$

$$\text{Co-efficient of sk} = \frac{3(20-17)}{0.04}$$

$$= \frac{3(3)}{0.04}$$

$$= \frac{9}{0.04}$$

Co-efficient Of sk = 2.25

---

2. For a moderatly skewed data, the arithmetic mean is 200, the co-efficient of variation is 8 and karl pearson's co-efficient of skewness is 0.3. Find the mode and median.

Sol:

$\bar{x} = 200$, $C.V = 8$, $Sk = 0.3$

$$C.V = \frac{SD \times 100}{mean}$$

$$8 = \frac{\sigma}{200} \times 100$$

$\sigma = 8 \times 2$

$\sigma = 16$

Coefficient Of $sk = \dfrac{\text{Mean} - \text{Mode}}{\sigma}$

$0.3 = \dfrac{200 - \text{mode}}{16}$

mode $= 200 - 4.8$

mode $= 195.2$

Mode $= 3(\text{median}) - 2(\bar{x})$

$195.2 = 3(\text{median}) - 2(200)$

$195.2 = 3(\text{median}) - 400$

$3\,\text{median} - 400 = 195.2$

$3\,\text{median} = 195.2 + 400$

median $= \dfrac{595.2}{3}$

median $= 198.4$

Median $= 198.4$ Mode $= 195.2$

3. For a group of 20 items, $\Sigma x = 1452$ and $\Sigma x^2 = 144280$ and mode $= 63.7$. Find the pearson's co-efficient of skewness.

Sol:-

$$Sk = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

$$\text{Mean} = \frac{\Sigma x}{N} = \frac{1452}{20} = 72.6$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

$$= \sqrt{\frac{144280}{20} - \left(\frac{1452}{20}\right)^2}$$

$$= \sqrt{7214 - 5270.76}$$

$$= \sqrt{1943.24}$$

$$\sigma = 44.08$$

$$Sk = \frac{72.6 - 63.7}{44.08}$$

$$= \frac{8.9}{44.08}$$

$$Sk = 0.202$$

Find the quartile co-efficient of skewness of the two groups gn below, which group is more skewness

Mark	A	B
55-58	12	20
58-61	17	22
61-64	23	25
64-67	18	13

Sol :

For group A :

C.I	f	cf
55-58	12	12 m_1
L_1 58-61	17 f_1	29 Q_1
61-64	23	52
64-67	18	70
	$\Sigma f = 70$	

$$Q_1 = L_1 + \frac{\frac{N}{4} - m_1}{f_1} \times c$$

$$= 58 + \frac{17.5 - 12}{17} \times 3$$

$$= 58 + \frac{5.5}{17} \times 3$$

$$= 58 + 0.9705$$

$$Q_1 = 58.9705$$

$$Q_2 = L_2 + \frac{\frac{N}{2} - m_2}{f_2} \times c$$

$$= 61 + \frac{35-29}{23} \times 3$$

$$= 61 + \frac{6}{23} \times 3$$

$$Q_2 = 61.78$$

$$Q_3 = L_3 + \frac{3\left(\frac{N}{4}\right) - m_3}{f_3} \times c$$

$$= 64 + \frac{3(17.5) - 52}{18} \times 3$$

$$= 64 + \cancel{8.749}\ 0.083$$

$$Q_3 = 64.083.$$

Bowley's coeff. of sk $= \dfrac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$

$$= \frac{64.083 + 58.97 - 2(61.78)}{64.083 - 58.97}$$

$$= \frac{123.053 - 123.56}{\ldots}$$

Bowley's co-efficient of skewness

$= -0.099.$

For group B:

C.I	f	cf
55-58	20	20 Q_1
58-61	22	42 Q_2
61-64	25	67 Q_3
64-67	13	80
	$\Sigma f = 80$	

$$Q_1 = L_1 + \frac{\frac{N}{4} - m_1}{f_1} \times c$$

$$= 55 + \frac{20 - 20}{20} \times 3$$

$$Q_1 = 55$$

$$Q_2 = L_2 + \frac{\frac{N}{2} - m_2}{f_2} \times c$$

$$= 58 + \frac{40 - 20}{22} \times 3$$

$$= 58 + \frac{20}{22} \times 3$$

$$= 58 + 2.727$$

$$Q_2 = 60.727$$

$$Q_3 = L_3 + \frac{3\left(\frac{N}{4}\right) - m_3}{f_3} \times c$$

$$= 61 + \frac{3(20) - 42}{25} \times 3$$

$$= 61 + \frac{18}{25} \times 3$$

$$= 61 + 2.16$$

$$Q_3 = 63.16$$

$$\text{Bowley's co-efficient of sk} = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{63.16 + 55 - 2(60.727)}{63.16 - 55}$$

$$= \frac{118.16 - 121.454}{8.16}$$

$$= \frac{-3.294}{8.16}$$

$$\left.\begin{array}{l}\text{Bowley's coefficient of} \\ \text{skewness}\end{array}\right\} = -0.4036$$

HomeWork:

1. Calculate karl pearson's coefficient of skewness from the gn data:

Income	400-500	500-600	600-700	700-800	800-900
No. of. employes	8	16	20	17	3

Sol:

i) Mean:

C.I	f	m	$d' = \frac{m-A}{c}$	fd'
400-500	8	450	-2	-16
500-600	16	550	-1	-16
600-700	20	650	0	0
700-800	17	750	1	17
800-900	3	850	2	6
	$\Sigma f = 64$			$\Sigma fd' = -9$

$$\text{Mean} = A \pm \frac{\Sigma fd'}{N} \times c$$

$$= 650 + \frac{(-9)}{64} \times 100$$

$$= 650 - 14.06$$

$$\text{Mean} = 635.94$$

Mode:

Grouping Table:

C.I	C_1	C_2	C_3	C_4	C_5	C_6
400 - 500	8	8+16 24				
500 - 600	16		16+20 36	8+16+20 44		
600 - 700	20	37			16+20+17 53	
700 - 800	17		20			20+17+3 40
800 - 900	3					

Analysis Table:

C.I	C_1	C_2	C_3	C_4	C_5	C_6	T
400 - 500				1			1
500 - 600			1	1	1		3
600 - 700	1	1	1	1	1	1	6
700 - 800		1			1	1	3
800 - 900						1	1

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times c$$

$f_0 = 16,\ f_1 = 20,\ f_2 = 17,\ L = 600$

$$= 600 + \left[\frac{20 - 16}{2(20) - 16 - 17}\right] \times 100$$

$$= 600 + 67.142$$

$$\text{Mode} = 657.142$$

$$S.D = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2}$$

C.I	f	m	$d' = \frac{m-A}{c}$	d'^2	fd'	fd'^2
400 - 500	8	450	-2	4	-16	32
500 - 600	16	550	-1	1	-16	16
600 - 700	20	650	0	0	0	0
700 - 800	17	750	1	1	17	17
800 - 900	3	850	2	4	6	12
	$\Sigma f = 64$				$\Sigma fd' = -9$	$\Sigma fd'^2 = 7$

$$S.D = \sqrt{\frac{77}{64} - \left(\frac{-9}{64}\right)^2}$$

$$= \sqrt{1.203 - (0.019)}$$

$$= \sqrt{1.184}$$

$$\boxed{SD = 1.088}$$

Difference b/w dispersion and skewness:

Dispersion	Skewness
* It shows us the spread of Individual values about the central value	* It shows us departure of symmetry
* It is useful to study the variablity in data.	* It is useful to study the concentration in lower or higher varaibles.
* It judges the truthfullness	* It judges the difference

of the central tendency	between the central tendency.
* It is a type of averages of deviation - Average of the second Order.	* It is not an average, but is measured by the use of the mean, the median and the mode
* It shows the degree of variability.	* It shows whether the concentration is in higher or lower value.

2. From the given data given below:

mean = 150, median = 142, $Q_1 = 62$, $Q_3 = 195$,

$d_1 = 30$, $d_9 = 230$, $\sigma = 30$

i) karl pearson's co-eff. of skewness

ii) Bowley's co-eff. of skewness

iii) ~~kaytey's~~ kelly's co-eff. of skewness

Sol:

iii) kelly's co-efficient of skewness:

gn: $d_1 = 30$, $d_9 = 230$, median = 142

$$= \frac{d_9 + d_1 - 2(\text{median})}{d_9 - d_1}$$

$$= \frac{230 + 30 - 2(142)}{230 - 30}$$

$$= \frac{230 + 30 - 284}{200}$$

$$= \frac{260 - 284}{200}$$

$$= -0.12$$

i) karl pearson's co-efficient of skewness:

$$= \frac{3(\text{mean} - \text{median})}{S.D}$$

$$= \frac{3(150-142)}{30}$$

$$= \frac{3(8)}{30} = \frac{24}{30}$$

$$= 0.8$$

iii) Bowley's co-efficient of skewness:

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{195 + 62 - 2(142)}{195 - 62}$$

$$= \frac{195 + 62 - 284}{133}$$

$$= \frac{-27}{133}$$

$$= -0.2030$$