MARUDHAR KESARI JAIN COLLEGE FOR WOMEN, VANIYAMBADI

PG & RESEARCH DEPARTMENT OF MATHEMATICS

CLASS : I M. C. A

SUBJECT CODE : 23PCA11

SUBJECT NAME : DISCRETE MATHEMATICS

SYLLABUS

UNIT 2

MATHEMATICAL LOGIC

Logical connectives-Well formed formulas Truth table of well formed formula Algebra of proposition - Normal forms of well formed formulas- Disjunctive normal form Principal Disjunctive normal form-Conjunctive normal form-Principal conjunctive normal form-Rules of Inference for propositional calculus Quantifiers- Universal Quantifiers- Existential Quantifiers.

UNIT-2

Mathematical logic:

Preposition:

A Preposition is a Statement which can be classified as true or false.

Eg:-

i) Chennai is the Capital of TN.
True

ii) What are you doing? (It is not Preposition)

iii) $A+B=C$ (Not Preposition)

iv) Chennai is Capital of kerala (False).

Section (A):

Logical Operators:-

1. Conjunction ($\wedge$):

Let p and q be two Prepositions it is denoted by $p \wedge q$. (P and q).

The Preposition $p \wedge q$ is Called Conjunction.

(i) If $p \wedge q$ is true when P and q are true.

(ii) If $p \wedge q$ is false when Otherwise P & q are true.

P	q	P^q
T	T	T
T	F	F
F	T	F
F	F	F

Eg:-

P: Today is wednesday

q: It is Sunny day

Sol:-

P^q : Today is wednesday and it is Sunny day.

2. **Disjunction: (V) (or)**

Let P and q are two Prepositions it is denoted by Pvq.

This Preposition Pvq is called disjunction.

(i) If Pvq is ~~true~~ false when P and q are false.

(ii) Pvq is true otherwise

P	q	Pvq
T	T	T
T	F	T
F	T	T
F	F	F

Eg:-

Pvq: Today is wednesday or it is Sunny day.

$\wedge$ T
$\vee$ F

3. Negation ($\sim$ / $\neg$):

Let P ~~and~~ q be the Preposition.
The Preposition $\sim P$ is called negation.

(i) If P is true then $\sim P$ is false.

(ii) If P is false then $\sim P$ is true.

P	$\sim P$
T	F
F	T

Eg:-

P: Today is wednesday

$\sim P$: Today is not wednesday.

if then

4. Implication / Conditional Statement : ($\rightarrow$)

Let P and q be a two Prepositions.
The Preposition $P \rightarrow q$ is Called implication.

(i) If $P \rightarrow q$ is ~~true~~ false when P is true & q is false.

(ii) If $P \rightarrow q$ is ~~false~~ true Otherwise

P	q	$\overset{T}{P} \rightarrow \overset{F}{q}$
T	T	~~F~~ T
T	F	F
F	T	~~F~~ T
F	F	~~F~~ T

Note :-

(i) P - hyphothesis

(ii) q - Conclusion

(iii) If P then q

(iv) P implies q

(v) P only if q

(vi) P is Sufficient for q

(vii) q if P

(viii) q whether P

(ix) q is necessary for P.

Eg:-

$p \to q$: If a man Study then he got Pass.

if and only if

5. BiConditional ($\leftrightarrow$) :

Let P and q be a two Preposition.

$P \leftrightarrow q$ is Called biConditional

(i) If $P \leftrightarrow q$ is true when P & q are Same.

(ii) If $P \leftrightarrow q$ is false otherwise.

P	q	$P \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Eg:-

$P \leftrightarrow q$: A man Study if and only if he Pass.

6. Converse :

Let P and q be a two Preposition.

$q \to P$ is Called Converse.

If (i) $q \to P$ is ~~true~~ false then q is true & P is false.

(ii) $q \to P$ is ~~false~~ true otherwise.

P	q	$\overset{T}{q} \to \overset{F}{P}$
T	T	~~F~~ T
T	F	~~F~~ T
F	T	~~T~~ F
F	F	~~F~~ T

7. Inverse :

Let P and q be the two Preposition and $\sim P, \sim q$ are negations, $P \to q$ is Conditional Preposition then $\sim P \to \sim q$ is Called Inverse of $P \to q$.

P	q	$\sim P$	$\sim q$	$\sim P \to \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

8. Contrapositive :

Let P and q be the two Preposition. Let ~P, ~q are negation

$\sim q \to \sim \cancel{q}\, p$ is Called Contrapositive of $P \to q$.

P	q	~P	~q	~q → ~P
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

$P \to q$ and $\sim q \to \sim p$ are logically equivalent.

P	q	~P	~q	P → q	~q → ~P
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

9. Tautology :

A Compound Preposition is always true is Called a tautology.

Eg :- Prove that $P \vee \sim P$ is a tautology.

p	~p	p∨~p
T	F	T
F	T	T

10. Contradiction:

A Compound Preposition is always false is called Contradiction.

Eg:- Prove that $p \wedge \sim p$ is Contradiction

p	~p	p∧~p
T	F	F
F	T	F

11. Contingency:

A Preposition that is neither a tautology nor a Contradiction is called Contingency.

Eg:-

p	p	p∨p
T	T	T
F	F	F

12. Logically Equivalent:

The Prepositions P and q are Called logically equivalent iff $P \leftrightarrow q$ is a tautology.

Eg :-

Prove that $\sim(P \wedge q)$ and $\sim P \vee \sim q$ are logically equivalent (or)

Prove that $\sim(P \wedge q) \equiv \sim P \vee \sim q$

P	q	$\sim P$	$\sim q$	$P \wedge q$	L.S $\sim(P \wedge q)$	R.S $\sim P \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Both L.S & R.S are equal.

$\sim(P \wedge q) \equiv \sim P \vee \sim q$

∴ Hence Proved.

13. Arguments :

An argument is an assertion that the given Set of Preposition $P_1, P_2 \ldots P_n$ yields q.

$P_1, P_2 \ldots P_n$ - Premices

q - Conclusion

The argument is denoted by $P_1, P_2 \ldots P_n \vdash q$ (The Symbol $\vdash$ is Called turnstile)

Note: 2

(i) If $P_1, P_2 \ldots P_n$ is true & q is true Contingency then $P_1, \ldots P_n \vdash q$ is true (valid arguments).

(ii) $P_1, P_2, \ldots P_n \vdash q$ is not true/false (i.e) neither true nor false (not valid argument / Fallacy) → False

Eg:-

Test the Validity of the argument

$P \leftrightarrow q, q \vdash P$.

Premises - $P \leftrightarrow q, q$

Conclusion - P

$P \leftrightarrow q, q \vdash P \Rightarrow (P \leftrightarrow q) \wedge q \rightarrow P$

P	q	$P \leftrightarrow q$	$(P \leftrightarrow q) \wedge q$	$(P \leftrightarrow q) \wedge q \rightarrow P$
T	T	T	T	T
T	F	F	F	T
F	T	F	F	T
F	F	T	F	T

H.W

Test $P \rightarrow q$ & $q \rightarrow P$ are logically equivalent

[redacted]

3. Verify whether $(P \wedge q) \wedge \sim(P \vee q)$ is a Contradiction.

P	q	$P \wedge q$	$P \vee q$	$\sim(P \vee q)$	$(P \wedge q) \wedge \sim(P \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

$(P \wedge q) \wedge \sim(P \vee q)$ is a Contradiction.

$\therefore$ Hence Verified

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Algebra of Proposition:

Proposition under the relation of logically equivalent.

(i) Idempotent law:

$$P \vee P \equiv P$$

$$P \wedge P \equiv P$$

(ii) Associative law:

$$P \vee (q \vee r) \equiv (P \vee q) \vee r$$

$$P \wedge (q \wedge r) \equiv (P \wedge q) \wedge r$$

(iii) Commutative law:

$$P \vee q \equiv q \vee P$$

$$P \wedge q \equiv q \wedge P$$

(iv) Distributive law:

$$P \vee (q \wedge r) \equiv (P \vee q) \wedge (P \vee r)$$

$$P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$$

(v) Demorgan's law:

$$\sim (P \vee q) \equiv \sim P \wedge \sim q$$

$$\sim (P \wedge q) \equiv \sim P \vee \sim q$$

(vi) Identity law:

$$P \vee f \equiv P$$

$$P \wedge t \equiv P$$

$$P \vee t \equiv t$$

$$P \wedge f \equiv f$$

(vii) Complement law :

$$p \vee \sim p \equiv t$$
$$p \wedge \sim p \equiv f$$
$$\sim(\sim p) \equiv p$$
$$\sim(t) \equiv f$$
$$\sim f \equiv t$$

Note :-

t and f are variables which are restricted to truth values of True & false respectively.

Prove that associative law for

(i) $p \vee (q \vee r) \equiv (p \vee q) \vee r$

p	q	r	$q \vee r$	$p \vee (q \vee r)$	$p \vee q$	$(p \vee q) \vee r$
T	T	T	T	T	T	T
T	~~F~~T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	F	T
F	F	F	F	F	F	F

L.H.S = R.H.S

Hence, $p \vee (q \vee r) \equiv (p \vee q) \vee r$

(ii) $P \wedge (q \wedge r) \equiv (P \wedge q) \wedge r$

P	q	r	$q \wedge r$	$P \wedge (q \wedge r)$	$P \wedge q$	$(P \wedge q) \wedge r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

L.H.S = R.H.S

Hence, $P \wedge (q \wedge r) \equiv (P \wedge q) \wedge r$

Prove that the Commutative law,

(i) $P \vee q \equiv q \vee P$

P	q	$P \vee q$	$q \vee P$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

L.H.S = R.H.S

Hence, $P \vee q \equiv q \vee P$

(ii) $P \wedge q \equiv q \wedge P$

P	q	$P \wedge q$	$q \wedge P$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

L.H.S = R.H.S

Hence, $P \wedge q \equiv q \wedge P$

Prove that the Distributive law,

(i) $P \vee (q \wedge r) \equiv (P \vee q) \wedge (P \vee r)$

P	q	r	$q \wedge r$	$P \vee (q \wedge r)$	$P \vee q$	$P \vee r$	$(P \vee q) \wedge (P \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T

[redacted]

(ii) $P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$

P	q	r	$q \vee r$	$P \wedge (q \vee r)$	$P \wedge q$	$P \wedge r$	$(P \wedge q) \vee (P \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

L.H.S = R.H.S

Hence, $P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$

Prove that the Demorgan's law,

(i) $\sim (P \vee q) \equiv \sim P \wedge \sim q$

P	q	$P \vee q$	$\sim (P \vee q)$	$\sim P$	$\sim q$	$\sim P \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

L.H.S = R.H.S

Hence, $\sim (P \vee q) \equiv \sim P \wedge \sim q$

(ii) $\sim(P \wedge q) \equiv \sim P \vee \sim q$

P	q	$P \wedge q$	$\sim(P \wedge q)$	$\sim P$	$\sim q$	$\sim P \vee \sim q$
T	T	T	F	F	F	F

[redacted]

(ii) $\sim (p \vee \sim r) \wedge (q \wedge \sim r)$

p	q	r	$\sim r$	$p \vee \sim r$	$\sim (p \vee \sim r)$	$q \wedge \sim r$	$\sim (p \vee \sim r) \wedge (q \wedge \sim r)$
T	T	T	F	T	F	F	F
T	T	F	T	T	F	T	F
T	F	T	F	T	F	F	F
T	F	F	T	T	F	F	F
F	T	T	F	F	T	F	F
F	T	F	T	T	F	T	F
F	F	T	F	F	T	F	F
F	F	F	T	T	F	F	F

1. Use the algebra of Preposition to Simplify the following.

(i) $\sim(\sim p \wedge \sim q)$ (ii) $\sim(\sim p \vee \sim q)$

(iii) $\sim(\sim p \vee q)$

[redacted]

2. Use the algebra of Preposition to Simplify the following.

$P \wedge (P \vee q) \equiv P$

Proof :-

$P \wedge (P \vee q) \equiv (P \wedge P) \vee (P \wedge q)$ ∵ Distributive

$\equiv P \vee (P \wedge q)$ ∵ Idempotent

$\equiv (P \wedge t) \vee (P \wedge q)$ ∵ Identity

$\equiv P \wedge (t \vee \cancel{qp} \wedge q$ ∵ distributive

$\equiv P \wedge t \wedge q$

$\equiv P \wedge (t \wedge q)$ ∵ $t \wedge q = t$

$\equiv P \wedge t$

$\equiv P$

$(P \wedge q) \vee \sim P \equiv \sim P \vee q$

$(P \wedge q) \vee \sim P \equiv \sim P \vee (P \wedge q)$ Commu

$\equiv (\sim P \vee P) \wedge (\sim P \vee q)$ distri

$\equiv T \wedge (\sim P \vee q)$ Complement

$\equiv \sim P \vee q$ Identity

H.W. $P \vee (P \wedge q) \equiv P$.

$P \vee (P \wedge q) \equiv (P \vee P) \wedge (P \vee q)$

$\equiv P \wedge (P \vee q)$

$\equiv (P \vee f) \wedge (P \vee q)$

$\equiv P \vee (f \wedge P) \vee q$

$\equiv P \vee f \vee q$

$\equiv P \vee (f \vee q)$ $\equiv P \vee f$ $\equiv P$

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2m

Well formed formula:

A Statement Variable, is Standing alone is Called Well formed formula (WFF).

(or)

A Statement formula is an expression which Consisting of Variable, Parenthesis and Connectors Symbols is Called Well formed formula.

Rules:

Eg:-

A Statement Variable Standing alone.

If P_i is WFF then negation P_i is also WFF.

If P and q are WFF then $(P \vee q)$, $(P \wedge q)$, $(P \to q)$, $(P \leftrightarrow q)$ is also WFF.

Not WFF:	WFF
$P \to q \to \wedge q$	$(P \to q) \to (P \wedge q)$
$\sim P \to q$	$(\sim P) \to q$ / $\sim (P \to q)$
$(P \to q)^{\gamma}$	$(P \to q)$

Normal Forms:

2m

Elementary Product:

A Product of Variables and their negations is Called an elementary Product.

2m

Elementary Sum:

A Sum of Variables and their negations is Called an elementary Sum.

Eg of elementary Product :

$P, \sim P, \sim P \wedge q, \sim P \wedge \sim q$

Eg of elementary Sum :

$P, \sim P, \sim P \vee q, \sim P \vee \sim q$

2m

Disjunctive normal form : (DNF)

A formula which is equivalent to a given formula and which consist of a <u>Sum of Elementary Products</u> is called a disjunctive normal form.

Eg :-

Product, Sum

$(P \wedge q) \vee r$

$(P \wedge q) \vee (P \wedge r)$

Note :-

Replace

1) $P \to q$ by $\sim P \vee q$

2) $P \leftrightarrow q$ by $(P \wedge q) \vee (\sim P \wedge \sim q)$

Steps :

i) use formula ① & ②

ii) Simplify negation

iii) Apply Demorgan's / distributive form.

1. Find DNF of the following :

(i) $P \wedge (P \to q)$

$$P \wedge (P \to q) = P \wedge (\sim P \vee q)$$
$$= (P \wedge \sim P) \vee (P \wedge q)$$

→ Final ans is in the form of DNF

(ii) $(P \to q) \wedge (\sim P \wedge q)$

$$(P \to q) \wedge (\sim P \wedge q) = (\sim P \vee q) \wedge (\sim P \wedge q)$$
$$= \sim P \wedge (\sim P \wedge q) \vee q \wedge (\sim P \wedge q)$$
$$= (\sim P \wedge q) \vee (\sim P \wedge q)$$

(∵ Similar → write 1 times)

$$= (\sim P \wedge q)$$

(iii) $\sim [P \to (q \wedge r)]$

$$\sim [P \to (q \wedge r)] = \sim [\sim P \vee (q \wedge r)]$$
$$= P \wedge \sim (q \wedge r)$$
$$= P \wedge (\sim q \overset{\vee}{\cancel{\wedge}} \sim r)$$
$$= (P \wedge \sim q) \overset{\vee}{\cancel{\wedge}} (P \wedge \sim r)$$

$\sim(\sim P) = P$

$\sim(\vee) = \wedge$

$\sim(\wedge) = \vee$

P q

(iv) $\sim (P \vee q) \leftrightarrow (P \wedge q)$

$$\sim (P \vee q) \leftrightarrow (P \wedge q) = [\sim (P \vee q) \wedge (P \wedge q)] \vee [\sim\sim (P \vee q) \wedge \sim (P \wedge q)]$$
$$= [(\sim P \overset{\wedge}{\cancel{\vee}} \sim q) \wedge (P \wedge q)] \vee [(P \vee q) \wedge (\sim P \vee \sim q)]$$
$$= [(\sim P \wedge P) \wedge (\sim q \wedge q)] \vee [P \wedge (\sim P \vee \sim q) \vee (q \wedge (\sim P \vee \sim q))]$$

$= (F \wedge F) \vee [[(P \wedge \sim P) \vee (P \wedge \sim q)] \vee$
$[(q \wedge \sim P) \vee (q \wedge \sim q)]]$

$= F \vee [F \vee (P \wedge \sim q) \vee (q \wedge \sim P) \vee F]$

$= F \vee [(P \wedge \sim q) \vee (q \wedge \sim P)]$

$= (P \wedge \sim q) \vee (q \wedge \sim P)$

2m.

Conjunctive Normal Form:

A formula which is Equivalent to a given formula which Consist of a Product of Elementary Sum is Called CNF.

Eg:-

$(P \vee q) \wedge r,$

$(P \vee q) \wedge (P \vee r)$

Note:

Replace:

1) $P \to q$ by $\sim P \vee q$

2) $P \leftrightarrow q$ by $(\sim P \vee q) \wedge (\sim q \vee P)$

Find CNF of the following

$P \wedge (P \to q)$

$P \wedge (P \to q) \equiv P \wedge (\sim P \vee q)$

(ii) $(\sim P \to r) \wedge (P \equiv q)$

$$(\sim P \to r) \wedge (P \equiv q) \equiv (\sim P \to r) \wedge (P \leftrightarrow q)$$

$$\equiv [\sim(\sim P) \vee r] \wedge [(\sim P \vee q) \wedge (\sim q \vee P)]$$

$$\equiv (P \vee r) \wedge (\sim P \vee q) \wedge (\sim q \vee P)$$

(iii) $\sim(P \vee q) \equiv (P \wedge q)$

$$\equiv \sim(P \vee q) \leftrightarrow (P \wedge q)$$

$$\equiv [\sim\sim(P \vee q) \vee (P \wedge q)] \wedge [\sim(P \wedge q) \vee \sim(P \vee q)]$$

$$\equiv [(P \vee q) \vee (P \wedge q)] \wedge [(\sim P \vee \sim q) \vee (\sim P \wedge \sim q)]$$

$$\equiv [P \vee (P \vee q) \wedge q \vee (P \vee q)] \wedge$$

$$[\sim P \vee (\sim P \vee \sim q) \wedge \sim q \vee (\sim P \vee \sim q)]$$

$$\equiv (P \vee q) \wedge (P \wedge q) \wedge (\sim P \vee \sim q) \wedge (\sim P \vee \sim q)$$

$$\equiv (P \vee q) \wedge (\sim P \vee \sim q)$$

2m

Minterm :

A minterm Consists of Conjunction.
If $P_1, P_2, \ldots P_n$ are n Variable then
$P_1 \wedge P_2 \wedge \ldots \wedge P_n$ is Said to be minterm.

Eg :-

$P \wedge q$, $P \wedge \sim q$, $\sim P \wedge q$, $\sim P \wedge \sim q$

2m

Maxterm :

A maxterm Consist of disjunctiveon.

If $P_1, P_2, \ldots P_n$ are n variables then $P_1 \vee P_2 \vee \ldots \vee P_n$ is said to be maxterm.

Eg :-

$P \vee q$, $P \vee \sim q$, $\sim P \vee q$, $\sim P \vee \sim q$

(7.) 2m

Principal Disjunctive Normal Form :

A formula Consisting of disjunctions of minterms only is known as PDNF.

(or)

$\vee$ Sum of minterms $\wedge$

Principal Conjunctive Normal Form :

A formula Consisting of Conjunctions of maxterms only is known as PCNF.

(or)

$\wedge$ Product of maxterms $\vee$

1. Obtain the principal disjunctive normal form & principal Conjunctive normal form of $P \leftrightarrow Q$.

T → P/Q F → ¬P/¬Q

P	Q	Minterm (∧)	P<->Q
T	T	$P \wedge Q$	T
T	F	$P \wedge \neg Q$	F
F	T	$\neg P \wedge Q$	F
F	F	$\neg P \wedge \neg Q$	T

PDNF : $P \leftrightarrow Q \rightarrow (P \wedge Q) \vee [\neg P \wedge \neg Q]$

Suppose $\neg(P \leftrightarrow Q)$ you write the minterm of false.

$\neg(P \leftrightarrow Q) \rightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$

PCNF : Write the false value with negation

$P \leftrightarrow Q \Rightarrow \sim[(P \wedge \sim Q) \vee (\sim P \wedge Q)]$

$= \sim(P \wedge \sim Q) \wedge \sim(\sim P \wedge Q)$

$= (\sim P \vee Q) \wedge (P \vee \sim Q)$

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2. Obtain the PDNF of

(i) $P \vee \sim q$

(ii) $P \to q$

(i) $P \vee \sim q$ Introduce some prepositions in 1st step.

$$P \vee \sim q = [P \wedge F] \vee [\sim q \wedge F]$$

$$= [P \wedge (q \vee \sim q)] \vee [\sim q \wedge (P \vee \sim P)]$$

$$= (P \wedge q) \vee (P \wedge \sim q) \vee (\sim q \wedge p) \vee (\sim q \wedge \sim p)$$

$$= (P \wedge q) \vee (P \wedge \sim q) \vee (\sim q \wedge P)$$

(ii) $P \to q$

$$\equiv \sim P \vee q$$

$$\equiv [\sim P \wedge (q \vee \sim q)] \vee [q \wedge (P \vee \sim P)]$$

$$\equiv (\sim P \wedge q) \vee (\sim P \wedge \sim q) \vee (q \wedge p) \vee (q \wedge \sim p)$$

$$\equiv (\sim P \wedge q) \vee (\sim P \wedge \sim q) \vee (q \wedge p)$$

3. Obtain the Conjunctive normal form of

(i) $P \wedge \sim q$

$$P \wedge \sim q \equiv [P \vee F] \wedge [\sim q \vee F]$$

$$\equiv [P \vee (q \wedge \sim q)] \wedge [\sim q \vee (P \wedge \sim p)]$$

$$\equiv [(P \vee q) \wedge (P \vee \sim q)] \wedge [(\sim q \vee p) \wedge (\sim q \vee \sim p)]$$

$$\equiv (P \vee q) \wedge (P \vee \sim q) \wedge (\sim q \wedge \sim p)$$

Quantifiers :- Represent Count.

Definition :

Certain Statements involve words that indicate quantity such as 'all', 'Some', 'none' or one. They answer the question 'How many?'.

Such words indicate quantify they are called quantifiers.

Eg :-

1) Some men are tall.
2) All birds have wings.

2) Universal quantifiers : (All)

The quantifiers 'all' is the universal quantifier. It is denoted by $\forall x$.

Same Meaning :

For all x

For every x

For each x

Everything x such that

Each thing x such that

3) Existential quantifiers :

The quantifiers 'Some' is existential quantifiers. It is denoted by $\exists x$.

Note:

For Some x

Some x Such that

There exists an, x Such that

There is an x Such that

There is atleast one x Such that

Note:

There are 2 quantifiers.

(i) $\forall$ - for every - universal quantifiers

(ii) $\exists$ - there Exist Some - Existence quantifiers.

Quantifiers with Single Predicate.

$(\forall x)\ P(x)$ eg: All Cows are black.

$(\exists x)\ P(x)$ eg: There Exist Some Cows which are black.

Quantifiers With binary Predicates:

$(\forall x)\ P(x,y) \Rightarrow \forall x \forall y\ [P(x,y)]$

$(\exists x)\ P(x,y) \Rightarrow \exists x \exists y\ [P(x,y)]$

Statement	Negation
$(\forall x)\ P(x)$	$(\exists x) \sim P(x)$
$(\forall x) \sim P(x)$	$(\exists x)\ P(x)$
$(\exists x)\ P(x)$	$(\forall x) \sim P(x)$
$(\exists x) \sim P(x)$	$(\forall x)\ P(x)$

Eg 1. If $A = \{1,2,3,4\}$. Find the truth values.

(i) $\exists x\,(x+5=15)$ where $x \in A$

False

(ii) $\forall x\,(x+3<10)$ where $x \in A$.

True

---

2. Symbolise the following

(i) For each real number x there exist another real number y such that $xy=1$.

$$\forall x\ \exists y \ni (xy=1) \qquad \cdot \exists \quad \ni$$

(ii) For all real number x & y $x.y = y.x$.

$\forall x$ & y

$$\forall x\ \forall y\quad x.y = y.x$$

(iii) There are real number x & y such that $x+y=1$

$$\exists x \ \& \ \exists y \ni x+y=1$$

Note :

$\forall x \forall y = \forall y \forall x$

$\exists x \exists y = \exists y \exists x$

3. Write in English $P(x)$: x is even

$R(x,y)$: $x+y$ is even

$Q(x)$: x is Prime

(i) $(\exists x)(\forall y)\ R(x,y)$

There exist x, for all y $x+y$ is even

(ii) $(\forall x)(\exists y)\ (R(x,y))$

For each x, there exist y $x+y$ is even.

(iii) $(\exists x) \sim P(x)$

There exist x is not even.

(iv) $(\forall x) \sim Q(x)$

For every x is not Prime.

4. Negate the following.

i) For all real number x if $x > 2$ then $x^2 > 4$

$P(x) = x > 2$

$Q(x) = x^2 > 4$

$\forall x, [P(x) \rightarrow Q(x)]$

$\forall x\ [\sim P(x) \vee Q(x)]$ $(P \rightarrow q) = (\sim P \vee q)$

Negating this now

$\exists x \sim [\sim P(x) \vee Q(x)]$

$\exists x \, [P(x) \wedge \sim Q(x)]$

There exist x $x > 2$ and $x^2 \ngtr 4$.

(ii) There is a real number x Such that if $x^3+y^2=3$ then $x>2$ & $y<5$

$P(x) = x^3+y^2=3$

$Q(x) = x>2$

$R(x) = y<5$

$\exists x \ni [P(x) \rightarrow (Q(x) \wedge R(x)]$

$\exists x \ni [\sim P(x) \vee (Q(x) \wedge R(x)]$

Negating it now

$\forall x \ni \sim[\sim P(x) \vee (Q(x) \wedge R(x)]$

$\forall x \ni [P(x) \wedge (\sim Q(x) \vee \sim R(x)]$

For each x Such that $x^3+y^2=3$ and $x \ngtr 2$ or $y \nless 5$.

(iii) For all x, if $x>3$ then $x^2>9$

$P(x) = x > 3$

$Q(x) = x^2>9$

$\forall x, [P(x) \rightarrow Q(x)]$

$\forall x, [\sim P(x) \vee Q(x)]$

Negating it now

$\exists x, \sim[\sim P(x) \vee Q(x)]$

$\exists x, [P(x) \wedge \sim Q(x)]$

There Exist x, $x>3$ and $x^2 \ngtr 9$.

5. Write each of the following in symbolic form.

(a) All men are giants

(b) No men are giants

(c) Some men are giants

(d) Some men are not giants

Sol:-

$P(x)$: x is a man

$Q(x)$: x is a giant.

(a) It meant for all x, if x is a man then x is giant.

$$\forall x\, [P(x) \rightarrow Q(x)]$$

(b) ($\forall x$, if x is a man, then x is not a giant)

$$\forall x\, [P(x) \rightarrow \sim Q(x)]$$

(c) (there is an x, such that x is a man & x is a giant)

$$\exists x\, [P(x) \wedge Q(x)]$$

(d) (there is an x, such that x is a man & x is not giant)

$$\exists x\, [P(x) \wedge \sim Q(x)]$$

6. Write the following Sentences in the Closed form. (Assume that the universe Consist of literally everything).

a) Some People who trust other are rewarded.

b) If any one is good then John is good.

c) He is ambitions or no one is ambitions.

d) Some one is testing.

e) It is not true that all roads lead to rome.

(a) Let $P(x)$: x is a Person

$T(x)$: x trusts others

$R(x)$: x is rewarded

(b) $G(x)$: x is good

(c) $A(x)$: x is ambitions

(d) $Q(x)$: x is testing

(e) $S(x)$: x is a Road

$L(x)$: x lead to rome

---

7. Bound occurence :

An Occurence of a Variable in a formula is Said to be bound occurence if this occurence is within the Scope of a quantifier using the Variable.

---

8. **Free Occurence :**

An occurence of a variable is called free occurence if this occurence of the variable is not a bound occurence.

9. **Free Variable :**

A variable is called free variable in a formula if atleast one occurence of the variable is a free occurence.

10. **Bound Variable :**

A variable is called bound variable in a formula if atleast one occurence of the variable is a bound occurence.

Theory of Inference:

If an implication $A \to B$ is a tautology, where A and B are Statement formulae, we Say that B logically follows from A, or B is a Valid Conclusion of the premise A. We Say that from a Set of Premise $\{H_1, H_2 \ldots H_n\} = C$. worked Examples:

W.E: 1

Show that $\neg Q, P \to Q \Rightarrow \neg P$

[1]	(1)	$P \to Q$	P
[1]	(2)	$\neg Q \to \neg P$	T(1) and E18
[3]	(3)	$\neg Q$	P
[1,3]	(4)	$\neg P$	T,(2),(3) and I_{11}.

W.E 2:

Show that $R \vee S$ is a Valid Conclusion from the premises $C \vee D$, $C \vee D \to \neg H$, $\neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to (R \vee S)$

Premises	Steps		
[1]	(1)	$C \vee D$	P
[2]	(2)	$(C \vee D) \to \neg H$	P
[1,2]	(3)	$\neg H$	T, (1),(2) and modules ponens. (I_{11})
[4]	(4)	$\neg H \to (A \wedge \neg B)$	P
[1,2,4]	(5)	$A \wedge \neg B$	T, (3), (4) and modus Ponens
[6]	(6)	$(A \wedge \neg B) \to (R \vee S)$	P
[1,2,4,6]	(7)	$R \vee S$	T, (5), (6) and modules ponens.

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Indirect method of Proof:

Introduce the negation of the desired Conclusion as a new premise.

From the new premise, together with the given premises, derive a Contradiction.

Assert the desired Conclusion as a logical inference from the premesis.

INDHO

W.E.1

Using indirect method of Proof, derive $P \to \neg S$ from $P \to Q \vee R$, $Q \to \neg P$, $S \to \neg R$, P.

The desired result is $P \to \neg S$. Its negation is $P \wedge S$. $(P \wedge S \leftrightarrow \neg(\neg P \vee \neg S) \leftrightarrow \neg(P \to \neg S)$ is a tautology. This follows from the law of negation for implication. We include $P \wedge S$ as an additional Premise.

[1]	(1)	$P \to Q \vee R$	P
[2]	(2)	P	P
[1,2]	(3)	$Q \vee R$	T(1), (2), modus ponens
[4]	(4)	$S \to \neg R$	P
[5]	(5)	$P \wedge S$	P (new premise)
[5]	(6)	S	T, (5) and Simplification
[1,5]	(7)	$\neg R$	T, (4), (6), modus ponens

[redacted]

a. Implications

I_1 $P \wedge Q \Rightarrow P$
I_2 $P \wedge Q \Rightarrow Q$ } (Simplification)

I_3 $P \Rightarrow P \vee Q$
I_4 $Q \Rightarrow P \vee Q$ } (addition)

I_5 $\neg P \Rightarrow P \rightarrow Q$

I_6 $Q \Rightarrow P \rightarrow Q$

I_7 $\neg (P \rightarrow Q) \Rightarrow P$

I_8 $\neg (P \rightarrow Q) \Rightarrow \neg Q$

I_9 $P, Q \Rightarrow P \wedge Q$

I_{10} $\neg P, P \vee Q \rightarrow Q$ (disjunctive Syllogism)

I_{11} $P, P \rightarrow Q \Rightarrow Q$ (modus ponens)

I_{12} $\neg Q, P \rightarrow Q \Rightarrow \neg P$ (modus tollens)

I_{13} $P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$ (hypothetical Syllogism)

I_{14} $P \vee Q, P \rightarrow R, Q \rightarrow R \Rightarrow R$ (dilemma)

I_{15} $P \rightarrow R, Q \rightarrow R \Rightarrow (P \vee Q) \rightarrow R$

b. Equivalences

$\neg\neg P \Leftrightarrow P$ (double negation)

$P \wedge Q \Leftrightarrow Q \wedge P$
$P \vee Q \Leftrightarrow Q \vee P$ } Commutative laws

$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$
$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$ } associative laws

$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ } distributive laws

$\neg(P\wedge Q) \Leftrightarrow \neg P \vee \neg Q$
$\neg(P\vee Q) \Leftrightarrow \neg P \wedge \neg Q$ } De morgan's laws

$P\vee P \Leftrightarrow P$

E_{11} $P\wedge P \Leftrightarrow P$

E_{12} $R\vee(P\wedge\neg P) \Leftrightarrow R$

E_{13} $R\wedge(P\vee\neg P) \Leftrightarrow R$

E_{14} $R\vee(P\vee\neg P) \Leftrightarrow T$

E_{15} $R\wedge(P\wedge\neg P) \Leftrightarrow F$

E_{16} $P\to Q \Leftrightarrow \neg P\vee Q$

E_{17} $\neg(P\to Q) \Leftrightarrow P\wedge\neg Q$

E_{18} $P\to Q \Leftrightarrow \neg Q\to\neg P$

E_{19} $P\to(Q\to R) \Leftrightarrow (P\wedge Q)\to R$

E_{20} $\neg(P\rightleftarrows Q) \Leftrightarrow P\rightleftarrows\neg Q$

E_{21} $(P\rightleftarrows Q) \Leftrightarrow (P\to Q)\wedge(Q\to P)$

E_{22} $(P\rightleftarrows Q) \Leftrightarrow (P\wedge Q)\vee(\neg P\wedge\neg Q)$

Example 1. Demonstrate that S is a valid inference from the $P\to\neg Q$, $Q\vee R$, $\neg S\to P$ and $\neg R$.

[1]	(1)	$Q\vee R$	premise (or hypothesis)
[2]	(2)	$\neg R$	premise (or hypothesis)
[1,2]	(3)	Q	(1), (2) and Tautology
[4]	(4)	$P\to\neg Q$	premise (or hypothesis)
[1,2,4]	(5)	$\neg P$	(3), (4) and tautology
[6]	(6)	$\neg S\to P$	premise (or hypothesis)
[1,2,4,6]	(7)	S	(5),(6) and tautology

Hence S is a valid inference.

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Quine's Method :-

	8	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

1. Find the minimal Sum of Products for the boolean expression.

$$Y(A,B,C,D) = \sum m\,(0,1,3,7,8,9,11,15)$$

using Quine's method.

Sol :-

Step 1 :

m	8 A	4 B	2 C	1 D
0	0	0	0	0
1	0	0	0	1
3	0	0	1	1
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
11	1	0	1	1
15	1	1	1	1

Step2 :

Group	Minterm	Binary relation ABCD	
0	m_0	0000	Zero
1	m_1 m_8	0001 1000	Single one
2	m_3 m_9	0011 1001	Double (1)
3	m_7 m_{11}	0111 1011	Triple (1)
4	m_{15}	1111	Four (1)

Step3 :

∵ [Changes → put (-)]
Comparing Single changes only taken

Group	Minterm	B.R ABCD
0	$m_0 - m_1$ $m_0 - m_8$	000– –000
1	$m_1 - m_3$ $m_1 - m_9$ $m_8 - m_9$	00–1 –001 100–
2	$m_3 - m_7$ $m_3 - m_{11}$ $m_9 - m_{11}$	0–11 –011 10–1
3	$m_7 - m_{15}$ $m_{11} - m_{15}$	–111 1–11

∴ [0 - put Compliment]

Step 4:

Group	Minterm	B.R ABCD	Prime Implicant
0	$(m_0 - m_1) - (m_8 - m_9)$	$-00-$	$\overline{B}.\overline{C}$
	$(m_0 - m_8) - (m_1 - m_9)$	$-00-$	
1	$(m_1 - m_3) - (m_9 - m_{11})$	$-0-1$	$\overline{B}.D$
	$(m_1 - m_9) - (m_3 - m_{11})$	$-0-1$	
2	$(m_3 - m_7) - (m_{11} - m_{15})$	$--11$	$C.D$
	$(m_3 - m_{11}) - (m_7 - m_{15})$	$--11$	

Prime Implicant	Minterm interval	0	1	3	7	8	9	11	15
$\overline{B}.\overline{C}$	0,1,8,9 ✓	×	×			×	×		
$\overline{B}.D$	1,3,9,11		×	×			×	×	
$C.D$	3,7,11,15 ✓			×	×			×	×

$$Y = \overline{B}.\overline{C} + C.D$$

[Note :- First check in Column wise, Circle the Single (x) Cross is present and then check the row where (x) Cross are Circle]