MARUDHAR KESARI JAIN COLLEGE FOR WOMEN, VANIYAMBADI

PG & RESEARCH DEPARTMENT OF MATHEMATICS

CLASS : I M. C. A

SUBJECT CODE : 23PCA11

SUBJECT NAME : DISCRETE MATHEMATICS

SYLLABUS

UNIT 4

MATRICES

Special types of matrices-Determinants-Inverse of a square matrix- -Elementary operations Rank of a matrix-Cramer's rule for solving linear equation -solving a system of linear equations-characteristic roots and characteristic vectors-Cayley-Hamilton Theorem-problems.

DISCRETE MATHEMATICS

UNIT- 4

MATRICES

MATRICES

DEFINITION: 2m

A Matrix is a rectangular array of numbers written in the form

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

Where a_{ij} are real or Complex number. They are Called as the elements of the matrix.

A matrix Containing m rows and n Columns is Said to be the Order $m \times n$.

Eg:-

$$\begin{bmatrix} 2 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}_{2 \times 3}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 7 & 8 \end{bmatrix}_{3 \times 2}$$

Types of Matrices:

2m

i) Row Matrix:

A matrix Containing only one row is Called a row matrix.

Eg:-

$$A = [a_{11} \; a_{12} \; \cdots \; a_{1n}]_{1 \times n}$$

2m

ii) Column Matrix:

A matrix Containing only one Column is Called a Column matrix.

Eg:-

$$A = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}_{m \times 1}$$

iii) Equal Matrix:

Two matrices A and B was Said to be equal if they are of the Same Order and the Corresponding elements are Equal

Eg:-

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$

$$B = \begin{bmatrix} 3 & 5 \\ 1 & 7 \end{bmatrix}_{2\times 2}$$

$a=3,\ b=5,\ c=1,\ d=7$

2m

iv) Square Matrix:

If $m=n$ (Number of rows = Number of Columns) then the given matrix is Called Square matrix.

Eg:-

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 4 \\ 2 & 1 & -3 \end{bmatrix}_{3\times 3}$$

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}_{2\times 2}$$

v) Diagonal Matrix:

A Square Matrix in which all the elements other then the leading diagonal are zero is called the diagonal Matrix.

Eg:-

$$A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2m.

vi) Scalar Matrix:

A diagonal Matrix in which all the diagonal elements are equal is called the Scalar Matrix.

Eg:-

$$A = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

vii) Unit Matrix:

A Scalar Matrix in which all the leading diagonal elements are unity is a unit matrix (or) unique matrix.

Eg:-

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

viii) Null Matrix :

* A Matrix in which all the elements are zero is called null matrix or zero matrix.

* Null matrix may be a Square matrix or rectangular matrix.

Eg:-

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}$$

bm

Matrix Operation:

Scalar Matrix:

When a matrix is multiplied by a Scalar every Elements in the matrix also multiplied by the Same Scalar

Eg:-

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$kA = \begin{bmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{32} & ka_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 4 \\ 6 & 1 \end{bmatrix}, \quad 3A = \begin{bmatrix} 6 & 12 \\ 18 & 3 \end{bmatrix}$$

Addition and Subtraction of Matrix:

Matrix Can be added only if they are of the Same Order

Eg:-

$$A+B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 4 & 7 \\ 2 & 5 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 0 & -1 \\ 0 & 3 \end{bmatrix}$$

Multiplication of Matrix:

i) (Two matrices A and B can be multiplied if only the number of column in A is equal to the number of row in B.) ii) (The Product matrix is denoted by AB.)

Eg:-

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}_{2\times 3} \quad B = \begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}_{3\times 2}$$

$$AB = \begin{bmatrix} 4+3+8 & 8+3+4 \\ 2+2+2 & 4+2+1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 15 & 15 \\ 6 & 7 \end{bmatrix}$$

Transpose of Matrix:

i) (For any given matrix A whose rows are columns of A and whose columns are rows of A, is called the transpose of A). ii) (It is denoted by A^T (or) A'.)

Eg:-

$$A = \begin{bmatrix} 4 & 6 & 8 \\ 3 & 2 & 1 \end{bmatrix}_{2\times 3}$$

$$A^T = \begin{bmatrix} 4 & 3 \\ 6 & 2 \\ 8 & 1 \end{bmatrix}_{3\times 2}$$

2m

Determinent of a matrix:

Consider the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

the determinent of the matrix is

$$|A| = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$|A| = a_{11}a_{22} - a_{21}a_{12}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$|A| = \begin{vmatrix} \overset{+}{a_{11}} & \overset{-}{a_{12}} & \overset{+}{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22}).$$

2m

Singular and Non-Sigular Matrix:

A Square matrix A is Said to be Sigigular if and only if is determinent is equal to zero.

i.e > $|A| = 0$

A Square matrix A is Said to be non-Singular if and only if is determinent is not equal to zero

i.e > $|A| \neq 0$

Scanned with ACE Scanner

Adjoint of a matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The adjoint of A is defined as to be the transpose of the Cofactor Matrix

$$\text{adj} = (A_{ij})^T$$

$$A_{ij} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$(A_{ij})^T = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

Reciprocal and inverse of a Matrix:

If A is non Singular matrix $A^{-1} = \frac{1}{|A|} \text{adj} A$ is defined be the inverse of the matrix. It is denoted by A^{-1} It Can be Show that $AA^{-1} = A^{-1}A = 1$ (or) I

(or)

A Square Matrix A of order n is Said to be invistible if there exists a Square matrix B of order n Such that $AB = BA = I_n$ and B is Called the inverse of A.

It is denoted by A^{-1}.

5m

Properties of determinent :

1. Let $A = [a_{ij}]$ be a $m \times n$ matrix then

i) if all the entries in a row (or Column) are zero then $|A| = 0$

ii) if there are two distinct values of i say s and r and a number α such that $a_{sj} = \alpha a_{rj}$ $\forall j = 1, 2 \ldots n$.

2. If A and B are Square matrices then $|AB| = |A||B|$

3. If A is a triangular matrix then $|A|$ is Product of the diagonal elements of A.

Problem :-

1. Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 1 \\ 3 & -5 \end{pmatrix}$ find AB.

$$AB = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 3 & -5 \end{pmatrix}$$

$$= \begin{pmatrix} 0-3 & 1+5 \\ 0+9 & 2-15 \end{pmatrix}$$

$$AB = \begin{pmatrix} -3 & 6 \\ 9 & -13 \end{pmatrix}$$

---

2. Find the adjoint of A. $A = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{pmatrix} 2 & 4 & -1 \\ 0 & 3 & 7 \\ 8 & 1 & 5 \end{pmatrix} \end{array}$

Cofactor of $2 = A_{11} = + \begin{vmatrix} 3 & 7 \\ 1 & 5 \end{vmatrix} = 15 - 7 = 8$

Cofactor of $4 = A_{12} = - \begin{vmatrix} 0 & 7 \\ 8 & 5 \end{vmatrix} = -(0 - 56) = 56$

$-1 = A_{13} = + \begin{vmatrix} 0 & 3 \\ 8 & 1 \end{vmatrix} = (0 - 24) = -24$

$0 = A_{21} = - \begin{vmatrix} 4 & -1 \\ 1 & 5 \end{vmatrix} = -(20 + 1) = -21$

$3 = A_{22} = + \begin{vmatrix} 2 & -1 \\ 8 & 5 \end{vmatrix} = (10 + 8) = 18$

$7 = A_{23} = - \begin{vmatrix} 2 & 4 \\ 8 & 1 \end{vmatrix} = -(2 - 32) = 30$

$8 = A_{31} = + \begin{vmatrix} 4 & -1 \\ 3 & 7 \end{vmatrix} = (28 + 3) = 31$

$1 = A_{32} = - \begin{vmatrix} 2 & -1 \\ 0 & 7 \end{vmatrix} = -(14 - 0) = -14$

$5 = A_{33} = + \begin{vmatrix} 2 & 4 \\ 0 & 3 \end{vmatrix} = (6 - 0) = 6$

Cofactor of $A = \begin{bmatrix} 8 & 56 & -24 \\ -21 & 18 & 30 \\ 31 & -14 & 6 \end{bmatrix}$

$\text{adj} A = A^T = \begin{bmatrix} 8 & -21 & 31 \\ 56 & 18 & -14 \\ -24 & 30 & 6 \end{bmatrix}$

Scanned with ACE Scanner

3. Find the inverse of A. $A = \begin{pmatrix} 2 & 4 & -1 \\ 0 & 3 & 7 \\ 8 & 1 & 5 \end{pmatrix}$

or if $A = \begin{pmatrix} 2 & 4 & -1 \\ 0 & 3 & 7 \\ 8 & 1 & 5 \end{pmatrix}$ find A^{-1}!

$$\boxed{A^{-1} = \frac{1}{|A|} \text{adj} A}$$

$$|A| = \begin{vmatrix} 2 & 4 & -1 \\ 0 & 3 & 7 \\ 8 & 1 & 5 \end{vmatrix}$$

$$= 2(15-7) - 4(0-56) - 1(0-24)$$

$$= 2(8) - 4(-56) - 1(-24)$$

$$= 16 + 224 + 24$$

$$|A| = 264$$

$$\begin{array}{r} 224 \\ 16 \\ 24 \\ \hline 264 \end{array}$$

$$A^{-1} = \frac{1}{264} \begin{bmatrix} 8 & -21 & 31 \\ 56 & 18 & -14 \\ -24 & 30 & 6 \end{bmatrix}$$

4. Find the inverse of $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

5. $$A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$|A| = \begin{vmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{vmatrix} \quad (-)$$

$$= 2(-4-1) - 3(-6-1) + 4(3-2)$$

$$= 2(-5) - 3(-7) + 4(1)$$

$$= -10 + 21 + 4$$

$$= 25 - 10$$

$$|A| = 15$$

$$\text{adj}A = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} +(-4-1) & -(-6-1) & +(3-2) \\ -(-6-4) & +(-4-4) & -(2-3) \\ +(3-8) & -(2-12) & +(4-9) \end{pmatrix}$$

$$= \begin{bmatrix} -5 & 7 & 1 \\ 10 & -8 & 1 \\ -5 & 10 & -5 \end{bmatrix}$$

$$\text{adj}A = A^T = \begin{bmatrix} -5 & 10 & -5 \\ 7 & -8 & 10 \\ 1 & 1\cancel{0} & -5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{15} \begin{bmatrix} -5 & 10 & -5 \\ 7 & -8 & 10 \\ 1 & 1 & -5 \end{bmatrix}$$

---

5. Find the adjoint of $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 2 & 5 \\ 4 & 1 & 0 \end{bmatrix}$.

$$\text{adj}A = \begin{bmatrix} +(0-5) & -\cancel{1}(0-20) & +\cancel{2}(2-8) \\ -(0-2) & +(0-8) & -(3-4) \\ +(5-4) & -(15-4) & +(6-2) \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 20 & -6 \\ 2 & -8 & 1 \\ 1 & -11 & 4 \end{bmatrix}$$

$$\text{adj}A = A^T = \begin{bmatrix} -5 & 2 & 1 \\ 20 & -8 & -11 \\ -6 & 1 & 4 \end{bmatrix}$$

6. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 2 & 1 \\ 4 & 6 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 4 & 7 \\ -2 & 3 & 8 \\ 6 & -3 & 4 \end{bmatrix}$

Show that $(A+B)^T = A^T + B^T$

$$A+B = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 2 & 1 \\ 4 & 6 & -5 \end{bmatrix} + \begin{bmatrix} 1 & 4 & 7 \\ -2 & 3 & 8 \\ 6 & -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 7 & 11 \\ 3 & 5 & 9 \\ 10 & 3 & -1 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 3 & 3 & 10 \\ 7 & 5 & 3 \\ 11 & 9 & -1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 2 & 5 & 4 \\ 3 & 2 & 6 \\ 4 & 1 & -5 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & -2 & 6 \\ 4 & 3 & -3 \\ 7 & 8 & 4 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 2 & 5 & 4 \\ 3 & 2 & 6 \\ 4 & 1 & -5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 6 \\ 4 & 3 & -3 \\ 7 & 8 & 4 \end{bmatrix}$$

$$A^T + B^T = \begin{bmatrix} 3 & 3 & 10 \\ 7 & 5 & 3 \\ 11 & 9 & -1 \end{bmatrix}$$

$$(A+B)^T = A^T + B^T$$

$\therefore$ Hence Proved.

Scanned with ACE Scanner

7. If $A = \begin{bmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$ Show that $(A-10I)(A-I)=0$ and find A^3

$$(A-10I)(A-I) = \left[\begin{pmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{pmatrix} - 10\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

$$\left[\begin{pmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

$$= \begin{pmatrix} 5-10 & 4-0 & -2-0 \\ 4-0 & 5-10 & -2-0 \\ -2-0 & -2-0 & 2-10 \end{pmatrix} \begin{pmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & 4 & -2 \\ 4 & -5 & -2 \\ -2 & -2 & -8 \end{pmatrix} \begin{pmatrix} 4 & 4 & -2 \\ 4 & 4 & -2 \\ -2 & -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -20+16+4 & -20+16+4 & 10-8-2 \\ 16-20+4 & 16-20+4 & -8+10-2 \\ -8-8+16 & -8-8+16 & 4+4-8 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(A-10I)(A-I) = 0$$

$$A^2 = \begin{pmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{pmatrix} \begin{pmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 25+16+4 & 20+20+4 & -10-8-4 \\ 20+20+4 & 16+25+4 & -8-10-4 \\ -10-8-4 & -8-10-4 & 4+4+4 \end{pmatrix}$$

$$= \begin{pmatrix} 45 & 44 & -22 \\ 44 & 45 & -22 \\ -22 & -22 & 12 \end{pmatrix}$$

$$A^2 . A^2 = \begin{pmatrix} 5 & 4 & -2 \\ 4 & 5 & -2 \\ -2 & -2 & 2 \end{pmatrix} \begin{pmatrix} 45 & 44 & -22 \\ 44 & 45 & -22 \\ -22 & -22 & 12 \end{pmatrix} \qquad \frac{44\times 5}{220} \quad \frac{45\times 4}{180}$$

$$= \begin{pmatrix} 225+176+44 & \overset{220+180+44}{\cancel{176+225+44}} & -110-\overset{88}{\cancel{110}}-\overset{24}{} \\ 180+220+44 & 176+225+44 & -88-110-24 \\ -90-88-44 & -88-90-44 & 44+44+24 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 445 & \overset{444}{\cancel{445}} & -\overset{222}{\cancel{244}} \\ 444 & 445 & -222 \\ -222 & -222 & 112 \end{pmatrix}$$

---

8. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix}$ find A^2, A^3

$$A^2 = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0+6 & 0+0+2 & 2+0+0 \\ 0+0+6 & 0+1+2 & 0+2+0 \\ 3+0+0 & 0+1+0 & 6+2+0 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 2 & 2 \\ 6 & 3 & 2 \\ 3 & 1 & 8 \end{pmatrix}$$

Scanned with ACE Scanner

$$A^3 = A \cdot A^2$$

$$= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{pmatrix} \begin{pmatrix} 7 & 2 & 2 \\ 6 & 3 & 2 \\ 3 & 1 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 7+0+6 & 2+0+2 & 2+0+16 \\ 0+6+6 & 0+3+2 & 0+2+16 \\ 21+6+0 & 6+3+0 & 6+2+0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 13 & 4 & 18 \\ 12 & 5 & 18 \\ 27 & 9 & 8 \end{pmatrix}$$

---

9. If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$ find A^2, A^3

$$A^2 = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 9-4 & -12+4 \\ 3-1 & -4+1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix}$$

$$A \cdot A^2 = \begin{pmatrix} 5 & -8 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 15-8 & -20+8 \\ 6-3 & -8+3 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 7 & -12 \\ 3 & -5 \end{pmatrix}$$

---

10. Find determinent of $A = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$

$$|A| = \begin{vmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

$$= 2(-4-1) - 3(-6-1) + 4(3-2)$$

$$= 2(-5) - 3(-7) + 4(1)$$

$$= -10 + 21 + 4$$

$$= -10 + 25$$

$$|A| = 15$$

11. Find determinent of $A = \begin{pmatrix} 2 & 4 \\ 1 & 1 \end{pmatrix}$

$$|A| = \begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix}$$

$$= 2 - 4$$

$$|A| = -2$$

Scanned with ACE Scanner

10 MARKS:-

1. Show that the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Statisfies the Equation. $A^3 - 6A^2 + 9A - 4I = 0$ and then deduce A^{-1}.

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A.A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -10-6-5 & 10+5+6 \\ -6-10-5 & 5+12+5 & -5-10-6 \\ 6+5+10 & -5-6-10 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$$A^3 - 6A^2 + 9A - 4I \neq \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6$$

$$-4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} + \begin{bmatrix} -36 & 30 & -30 \\ 30 & -36 & 30 \\ -30 & 30 & -36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix}$$

$$+ \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36+18-4 & -21+30-9+0 & 21-30+9+0 \\ -21+30-9+0 & 22-36+18-4 & -21+30-9+0 \\ 21-30+9+0 & -21+30-9+0 & 22-36+18-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 - 6A^2 + 9A - 4I = 0$$

Deduce A^{-1}.

$$A^3 - 6A^2 + 9A - 4I = 0$$

$\times^{ly}$ A^{-1}

$$A^{-1}A^3 - 6A^{-1}A^2 + 9AA^{-1} - 4A^{-1}I = 0.$$

$$A^2 - 6A + 9I - 4A^{-1}(1) = 0$$

Both S $\times$ by(-)

$$-4A^{-1} = -A^2 + 6A - 9I$$

$$4A^{-1} = A^2 - 6A + 9I$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} +$$

$$9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix}$$

Scanned with ACE Scanner

$$4A^{-1} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

---

2) Show that the matrix $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Statisfies the Equation $A^2 - 4A + 3I = 0$
& then deduce A^{-1}.

$$A^2 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & -2-2 \\ -2-2 & 1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix}$$

$$A^2 - 4A + 3I = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} - 4\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5-8+3 & -4+4+0 \\ -4+4+0 & 5-8+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 - 4A + 3I = 0$$

$$A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$= \frac{1}{}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= 4 - 1$$

$$= 3$$

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Or

Deduce A^{-1}

$\times$ by A^{-1}

$$A^2A^{-1} - 4AA^{-1} + 3IA^{-1} = 0$$

$$A - 4I + 3A^{-1}(1) = 0$$

$$+3A^{-1} = -A + 4I$$

$\times$ by $(-)$

$$-3A^{-1} = A - 4I$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4 & -1+0 \\ -1-0 & 2-4 \end{bmatrix}$$

$$-3A^{-1} = \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$$

$$+A^{-1} = \frac{1}{-3} \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Symmentric and Skew Symmetric:

A Square matrix $A = [a_{ij}]$ is called Symmetric matrix if the $(i,j)^{th}$ element of A is equal to the $(j,i)^{th}$ element of A.

$$(i.e)\ [a_{ij}] = [a_{ji}]\ \forall i,j$$

$$\boxed{A = A'}$$

A Square matrix $A = [a_{ij}]$ is said to be Skew Symmetric if the $(i,j)^{th}$ element is equal to the negative ~~to~~ of th $(j,i)^{th}$ element of A.

$$(i.e)\ [a_{ij}] = -\cancel{a}[a_{ji}]\ \forall i,j$$

Theorem:

$$\boxed{A = -A'}$$

Show that every Square matrix Can be uniquely Expressed as the Sum of Symmetric and Skew Symmetric.

Let A be a Square matrix.

Therefore $A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A') \rightarrow (1)$

$A = A + A'$

Now $\left(\frac{A+A'}{A'}\right)' = A' + (A')' = A' + A = A + A'$

$=$ A

$(A-A')' = A' - (A')' = A' - A = -(A-A')$

$A' = -A$

$A+A'$ is Symmetric

$A-A'$ is Skew Symmetric

from ①

$$A = P + Q \rightarrow ②$$

$P = \frac{1}{2}(A+A')$ is Symmetric

$Q = \frac{1}{2}(A-A')$ is Skew Symmetric

Any Square matrix Can be Expressed as the Sum of Symmetric and Skew Symmetric.

Uniqueness :

Suppose $A = R+S \rightarrow ③$

Where R is Symmetric

S is Skew Symmetric

Then

$$A = R+S$$

$$A' = R-S \rightarrow ④$$

③ & ④ $A + A' = 2R$

$$R = \frac{1}{2}(A+A') = P$$

$$A - A' = 2S$$

$$S = \frac{1}{2}(A-A') = Q$$

Therefore there is only one way of Expressing a Square matrix as a Sum of Symmetric and Skew Symmetric.

Express $A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$ as the Sum of Symmetric and Skew Symmetric.

$$A' = \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}$$

$$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$$

$$\frac{1}{2}(A+A') = \frac{1}{2}\left\{\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 12 & 12 & 14 \\ 12 & 4 & 10 \\ 14 & 10 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix}$$

$$\frac{1}{2}(A-A') = \frac{1}{2}\left\{\begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 9 \\ 8 & 2 & 7 \\ 5 & 3 & 1 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 4 & -4 \\ -4 & 0 & -4 \\ 4 & 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & -2 \\ 2 & 2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & 6 & 7 \\ 6 & 2 & 5 \\ 7 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & -2 \\ 2 & 2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & 8 & 5 \\ 4 & 2 & 3 \\ 9 & 7 & 1 \end{bmatrix}$$

Theorem: If A & B are both Symmetric then $A \cdot B$ is Symmetric if and only if A and B are Commutative.

Proof:- Since A & B is Symmetric

$$A' = A$$

$$B' = B$$

Hence $(AB)' = B'A' = BA \rightarrow ①$

$\Rightarrow$ ① If A & B are Commutative then

$$AB = BA \rightarrow ②$$

from ① $\Rightarrow (AB)' = BA = AB$

AB is Symmetric

$\Rightarrow$ ② Also (AB) is Symmetric

Then $(AB)' = AB \rightarrow ③$

But $(AB)' = B'A' = BA \rightarrow ④$

from ③ & ④

$$AB = BA$$

Hence A & B is Commutative.

Scanned with ACE Scanner

Theorem :

If A & B are Symmetric (Skew Symmetric) Show that $A+B$ is Symmetric (Skew Symmetric)

i) A & B are Symmetric

$$A' = A$$

$$B' = B$$

Now $(A+B)' = A' + B' = \underline{A+B}$

$A+B$ is Symmetric.

ii) A & B is Skew Symmetric

$$A' = -A$$

$$B' = -B$$

$$(A+B)' = A' + B' = -A - B = -(A+B)$$

$A+B$ is Skew Symmetric.

10m

Cramer's Rule :

Consider the Equation

$$a_1 x + b_1 y + c_1 z = d_1 \rightarrow ①$$

$$a_2 x + b_2 y + c_2 z = d_2 \rightarrow ②$$

$$a_3 x + b_3 y + c_3 z = d_3 \rightarrow ③$$

Δ

Let $\Delta = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

$\times$lying both Side by x

$$\Delta x = \begin{bmatrix} a_1 x & b_1 & c_1 \\ a_2 x & b_2 & c_2 \\ a_3 x & b_3 & c_3 \end{bmatrix}$$

$$\begin{bmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{bmatrix} = \Delta x \text{ (Say)}$$

- Then $x = \frac{\Delta x}{\cancel{x}\Delta}$ $x = \frac{\Delta x}{\Delta}$

Similarly $y = \frac{\Delta y}{\Delta}$

$$z = \frac{\Delta z}{\Delta}$$

Where

$$\Delta y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix} \quad \Delta z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

1. Solve the equation

$$x + y + z = -1 \rightarrow (1)$$

$$x + 2y + 3z = -4 \rightarrow (2)$$

$$x + 3y + 4z = -6 \rightarrow (3)$$

Let $\Delta = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

$$= 1(8-9) - 1(4-3) + 1(3-2)$$

$$= -1 - 1 + 1$$

$$\boxed{\Delta = -1}$$

$$\Delta x = \begin{bmatrix} -1 & 1 & 1 \\ -4 & 2 & 3 \\ -6 & 3 & 4 \end{bmatrix}$$

$$= -1(8-9) - 1(-16+18) + 1(-12+12)$$

$$= -1(-1) - 1(2) + 1(0)$$

$$= 1 - 2 + 0$$

$$\boxed{\Delta x = -1}$$

$$\Delta y = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -4 & 3 \\ 1 & -6 & 4 \end{bmatrix}$$

$$= 1(-16+18) + 1(4-3) + 1(-6+4)$$

$$= 1(2) + 1(1) + 1(-2)$$

$$= 2+1-2$$

$$\boxed{\Delta y = 1}$$

$$\Delta z = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -4 \\ 1 & 3 & -6 \end{bmatrix}$$

$$= 1(-12+12) - 1(-6+4) - 1(3-2)$$

$$= 1(0) - 1(-2) - 1(1)$$

$$= 0+2-1$$

$$\boxed{\Delta z = 1}$$

$$x = \frac{\Delta x}{\Delta} = \frac{-1}{-1} = 1$$

$$y = \frac{\Delta y}{\Delta} = \frac{1}{-1} = -1$$

$$z = \frac{\Delta z}{\Delta} = \frac{1}{-1} = -1$$

$\therefore \boxed{x = 1}$

$\therefore \boxed{y = -1}$

$\therefore \boxed{z = -1}$

H.W

2. Solve the equation

$$2y - 3z = 0 \rightarrow ①$$

$$x + 3y = -4 \rightarrow ②$$

$$3x + 4y = 3 \rightarrow ③$$

Let $\Delta = \begin{bmatrix} 0 & 2 & -3 \\ 1 & 3 & 0 \\ 3 & 4 & 0 \end{bmatrix}$

$= 0(0-0) - 2(0-0) - 3(4-9)$

$= 0 - 0 - 3(-5)$

$\Delta = 15$

$\Delta x = \begin{bmatrix} 0 & 2 & -3 \\ -4 & 3 & 0 \\ 3 & 4 & 0 \end{bmatrix}$

$= 0(0-0) - 2(0-0) - 3(-16-9)$

$= 0-0-3(-25)$

$= 75$

$\Delta y = \begin{bmatrix} 0 & 0 & -3 \\ 1 & -4 & 0 \\ 3 & 3 & 0 \end{bmatrix}$

$= 0(-0-0) - 0 - 3(3+12)$

$= -3(15)$

$= -45$

$\Delta z = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 3 & -4 \\ 3 & 4 & 3 \end{vmatrix}$

$= 0 - 2(3+12) + 0$

$= -2(15)$

$= -\cancel{45}\,30$

$x = \frac{\Delta x}{\Delta} = \frac{75}{15} = 5$

$\boxed{x = 5}$

$y = \frac{\Delta y}{\Delta} = \frac{-45}{15} = -3$

$\boxed{y = -3}$

$z = \frac{\Delta z}{\Delta} = \frac{-30}{15} = -2$

$\boxed{z = -2}$

Scanned with ACE Scanner

3. Solve the equation

$$\frac{x^2 z^3}{y} = e^8$$

$$\frac{y^2 z}{x} = e^4$$

$$\frac{x^3 y}{z^4} = 1$$

log 0 = 1
log 1 = 0

Formula :-

$\log\left(\frac{a}{b}\right) = \log a - \log b$

$\log(ab) = \log a + \log b$

$\log a^x = x \log a$

Taking log on both Side

$$\log\left(\frac{x^2 z^3}{y}\right) = \log(e^8)$$

$$\log(x^2 z^3) - \log y = 8$$

$$\log x^2 + \log z^3 - \log y = 8$$

$$2\log x + 3\log z - \log y = 8$$

$$2\log x - \log y + 3\log z = 8 \rightarrow (1)$$

Taking log on b.S

$$\log\left(\frac{y^2 z}{x}\right) = \log(e^4)$$

$$\log(y^2 z) - \log x = 4$$

$$\log y^2 + \log z - \log x = 4$$

$$2\log y + \log z - \log x = 4$$

$$-\log x + 2\log y + \log z = 4 \rightarrow (2)$$

$$\frac{x^3 y}{z^4} = 1$$

Taking log on b.S

$$\log\left(\frac{x^3 y}{z^4}\right) = \log 1$$

$$\log(x^3 y) - \log z^4 = 0$$

$$\log x^3 + \log y - \log z^4 = 0$$

$$3\log x + \log y - 4\log z = 0 \rightarrow (3)$$

This is a Set of linear equation in the variables $\log x, \log y, \log z$

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{vmatrix}$$

$$= 2(-8-1) + 1(4-3) + 3(-1-6)$$

$$= 2(-9) + 1(1) + 3(-7)$$

$$= -18 + 1 - 21$$

$$= -39 + 1$$

$$\Delta = -38$$

$$\Delta \log x = \begin{vmatrix} 8 & -1 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -4 \end{vmatrix}$$

$$= 8(-8-1) + 1(-16-0) + 3(4-0)$$

$$= 8(-9) + 1(-16) + 3(4)$$

$$= -72 - 16 + 12$$

$$= -88 + 12$$

$$= -76$$

$$\Delta \log y = \begin{vmatrix} 2 & 8 & 3 \\ -1 & 4 & 1 \\ 3 & 0 & -4 \end{vmatrix}$$

$$= 2(-16-0) - 8(4-3) + 3(0-12)$$

$$= 2(-16) - 8(1) + 3(-12)$$

$$= -32 - 8 - 36$$

$$= -76$$

$$\Delta \log z = \begin{vmatrix} 2 & -1 & 8 \\ -1 & 2 & 4 \\ 3 & 1 & 0 \end{vmatrix}$$

$$= 2(0-4) + 1(-0-12) + 8(-1-6)$$

$$= 2(-4) + 1(-12) + 8(-7)$$

$$= -8 - 12 - 56$$

$$\Delta \log z = -76$$

$$\log x = \frac{\Delta \log x}{\Delta}$$

$$= \frac{-76}{-38}$$

$$\log x = 2$$

Taking 'e' on b.S

$$\log \cancel{e}^{x} = e^{2}$$

$$\therefore \boxed{x = e^{2}}$$

$$\log y = \frac{\Delta \log y}{\Delta} = \frac{-76}{-38} = 2$$

Taking 'e' on b.S

$$\cancel{e}^{\log y} = e^{2}$$

$$\therefore \boxed{y = e^{2}}$$

$$\log z = \frac{\Delta \log z}{\Delta} = \frac{-76}{-38} = 2$$

Taking 'e' on b.S

$$\cancel{e}^{\log z} = e^{2}$$

$$\therefore \boxed{z = e^{2}}$$

Elementary Operation & Rank of Matrix:

There are three types of elementary row operation and three types of elementary column operation.

i) The interchange of any two rows.

ii) Multiplying multiplication of a row by a non zero number (α).

iii) Addition of any multiple of one row with any other row.

Inverse of a matrix using row Operation:

Find the inverse of $\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix}$

using elementary operation.

$$|A| = \begin{vmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{vmatrix}$$

$$= 8(-4+2) + 1(20-20) - 3(5-10)$$

$$= 8(-2) + 0 - 3(-5)$$

$$= -16 + 15$$

$|A| \neq -1$ $\quad |A| \neq 0$ ✓

The inverse A^{-1} exist

$$\boxed{A = IA}$$

$$\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & \frac{-1}{8} & -3/8 \\ -5 & 1 & 2 \\ 10 & -1 & -4 \end{bmatrix} = \begin{bmatrix} 1/8 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad R_1 \to \frac{R_1}{8}$$

$$\begin{bmatrix} 1 & -1/8 & -3/8 \\ 0 & 3/8 & 1/8 \\ 0 & 2/8 & -2/8 \end{bmatrix} = \begin{bmatrix} 1/8 & 0 & 0 \\ 5/8 & 1 & 0 \\ -10/8 & 0 & 1 \end{bmatrix} A \quad \begin{matrix} R_2 \to R_2 + 5R_1 \\ R_3 \to R_3 - 10R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & -1/8 & -3/8 \\ 0 & 1 & 1/3 \\ 0 & 2/8 & -2/8 \end{bmatrix} = \begin{bmatrix} 1/8 & 0 & 0 \\ 5/8 & 8/3 & 0 \\ -10/8 & 0 & 1 \end{bmatrix} A \quad R_2 \to \left(\frac{8R_2}{3}\right)$$

$$\begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 1/3 \\ 0 & 0 & -1/3 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 0 \\ 5/3 & 8/3 & 0 \\ -5/3 & -2/3 & 1 \end{bmatrix} A \quad \begin{matrix} R_1 \to R_1 + \frac{1}{8}R_2 \\ R_3 \to R_3 - \frac{2}{8}R_2 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 1/3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 0 \\ 5/3 & 8/3 & 0 \\ 5 & 2 & -3 \end{bmatrix} A \quad R_3 \to -3R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix} A \quad \begin{matrix} R_1 \to R_1 + \frac{1}{3}R_3 \\ R_2 \to R_2 - \frac{1}{3}R_3 \end{matrix}$$

Hence $A^{-1} = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$

Rank of Matrix :-

Two matrices A and B of the Same Order are Said to be equivalent to each other if one of them can be obtained from the other by Successive applications of elementary row and Column operations. We write,

$$A \sim B$$

Canonical form

If A is a $m \times n$ matrix then the unique non negative integer 'r' Such that

$$A \sim \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$ is Said to be the rank of A. and is denoted by $r(A)$. The matrix is Called Canonical form of A.

Find the rank of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 4 & 2 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 4 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -3 & -4 & -2 \\ 0 & 3 & 4 & 2 \end{bmatrix} \quad R_2 \to R_2 - 4R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -3 & -4 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 4/3 & 2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 \to -\frac{1}{3} R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4/3 & 2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \\ C_4 \to C_4 - C_1 \end{matrix}$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{matrix} C_3 \to C_3 - \frac{4}{3} C_2 \\ C_4 \to C_4 - \frac{2}{3} C_2 \end{matrix}$$

$$\sim \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$\sigma(A) = 2$

Find the Canonical form of $A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 1 & 2 & 8 \end{bmatrix} \quad R_3 \to \frac{R_3}{3}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 \to R_1 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C_3 \to C_3 + C_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C_3 \to C_4 - 2C_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C_4 \to C_4 + 2C_1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C_4 \to C_4 - 8C_2$$

$$\sim \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2$$

Solving a System of linear Equation

(A System $Ax = B$ Suppose we are given m equation and) n)

Consider a System of m linear Equations in n variables $x_1, x_2 \ldots x_n$ given by

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$
$$\ldots$$
$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m$$

Here the ~~Cartesians~~ Coefficients a_{ij} are real or Complex numbers. The Constants $b_1, b_2 \ldots b_m$ are also real or Complex numbers.

The System Can be written as $Ax = B$ Where

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix},$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ and } B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

The $m \times n$ matrix A is called the Coefficient matrix.

The $m \times (n+1)$ matrix $\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} & b_1 \\ a_{21} & a_{22} & \ldots & a_{2n} & b_2 \\ \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} & b_m \end{bmatrix}$

is called the argumented matrix of the System & is denoted by $[A|B]$

As A is a Submatrix of a matrix [A|B].

We have rank of A $R(A) \leq R[A|B]$

The System have a Solution

A System $Ax=B$ is Said to be Consistent if it has atleast one Solution otherwise it is Said to be inConsistent.

i) A System $A=B$ is Consistent if and only if $R(A) = R[A|B]$ Solution

ii) Has a Unique Solution if and only if rank of A = $R(A) < R(A,B)$

$R(A) = R[A|B]$ InConsistent & no Sol

$= \min\{m,n\}$

iii) As infinitely many Solution if and only if $R(A) = R[A|B]$

$< \min\{m,n\}$

i) Coefficient of A
ii) argument (A,B)
iii) find the Rank

Consistent

$R(A) = R.R(A|B) = r$

$r = n$ Unique

$R(A) = R(A,B) = r$

$r < n$ infinite

Verify whether the following System of equation is Consistent if it is Consistent find the Solution.

$$x_1 + x_2 + 2x_3 = 1$$

$$2x_1 + x_2 + 4x_3 = 2$$

$$3x_1 + 5x_2 + x_3 = -1$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 4 \\ 3 & 5 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$[A,B] = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 4 & 2 \\ 3 & 5 & 1 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -1 & 0 & 0 \\ 3 & 5 & 1 & -1 \end{bmatrix} \quad R_2 \to R_2 - 2R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 2 & -5 & -4 \end{bmatrix} \quad R_3 \to R_3 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -5 & -4 \end{bmatrix} \quad R_3 \to R_3 + 2R_2$$

$\rho(A) = 3$

$\rho(A,B) = 3$

$\rho(A) = \rho(A,B)$

The System of equation is Consistent.

$$x_1 + x_2 + 2x_3 = 1$$

$$0x_1 - x_2 + 0x_3 = 0$$

$$0x_1 + 0x_2 - 5x_3 = -4$$

$\boxed{x_2 = 0}$

$-5x_3 = -4$

$\boxed{x_3 = \frac{4}{5}}$

$x_1 + x_2 + 2x_3 = 1$

$x_1 + 0 + 2\left(\frac{4}{5}\right) = 1$

$x_1 + \frac{8}{5} = 1$

$x_1 = 1 - \frac{8}{5}$

$= \frac{5-8}{5}$

$\boxed{x_1 = \frac{-3}{5}}$

The Solutions are

$x_1 = -3/5$

$x_2 = 0$

$x_3 = 4/5$

---

2. $x + 2y + z = 11$

$4x + 6y + 5z = 8$

$4x + 4y + 6z = 38$

$A = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 6 & 5 \\ 4 & 4 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 11 \\ 8 \\ 38 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$[A, B] = \begin{bmatrix} 1 & 2 & 1 & 11 \\ 4 & 6 & 5 & 8 \\ 4 & 4 & 6 & 38 \end{bmatrix}$

Scanned with ACE Scanner

$$\sim \begin{bmatrix} 1 & 2 & 1 & 11 \\ 0 & -2 & 1 & -36 \\ 0 & -4 & 2 & -6 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 4R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 11 \\ 0 & -2 & 1 & -36 \\ 0 & 0 & 0 & 66 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$\rho(A) = 2$

$\rho(A, B) = 3$

$\rho(A) \neq \rho(A, B)$

The System of equation is inconsistent & there is no Solution.

---

3 For what values of λ & μ the System of equation

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

i) Inconsistent

ii) Consistent

iii) Consistent and the Solution is unique.

iv) Infinite Number of Solution

$$[A, B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$\rho(A) = 3$

$\rho(A,B) = 3$

$\rho(A) = \rho(A,B) = 3$

(i) Inconsistent:

If $\lambda = 3$ and $\mu \neq 10$ $\quad \rho(A) \neq \rho(A,B)$

The System is inconsistent.

(ii) Consistent:

If $\lambda \neq 3$ and $\mu \neq 10$

$\rho(A) = 3, \ \rho(A,B) = 3$

$\therefore$ The System of Equation is Consistent

(iii) Consistent and the Solution is unique:

If $\lambda \neq 3$ & $\mu \neq 10$

$\rho(A) = 3 \quad \rho(A,B) = 3$

$r = 3$ & $n = 3$

$r = n$

$\therefore$ The System of equation is Consistent and has unique Solution.

(iv) Infinite Number of Solution:

If $\lambda = 3$ & $\mu = 10$ $\quad \rho(A) = 2 \quad \rho(A,B) = 2$

$r = 2$ & $n = 3$

$r < n$

$\therefore$ The System of Equation has inconsistent & has infinitly many Solution

Eigen Values and Eigen Vectors :

1. A Square Matrix A and its transpose A^T have the Same eigen Values.

2. If a Square matrix is a triangular matrix, its Characteristic roots are diagonal elements. In Particular for a diagonal matrix, its diagonal elements on the characteristic roots.

3. The Sum of the eigen Values of a matrix A is equal to the Sum of elements on its diagonal.

The Sum of diagonal elements of a Square matrix A is Called in trace.

Hence we have the result

The Sum of the eigen Values of a matrix is equal to its trace. Hence we have the result.

4. If $\lambda_1, \lambda_2, \ldots \lambda_n$ are the eigen Values of a matrix A.

i) the inverse matrix A^{-1} has the eigen Values $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots \frac{1}{\lambda_n}$.

ii) the matrix A^m (where m is a Positive integer) has the eigen values $\lambda_1^m, \lambda_2^m, \ldots \lambda_n^m$.

iii) The matrix kA (where k is an arbitrary Scalar) has the latent roots $k\lambda_1, k\lambda_2, \ldots k\lambda_n$. (Eigen values are also Called latent roots).

5. Every Square matrix A Satisfies its own characteristic equation. (Cayley Hamilton theorem).

6. The Characteristic roots of a real Symmetric matrix are all real.

Theorem:

Two Similar matrices have the Same Characteristic roots.

Proof:

Let A and B be Similar matrix, then $B = P^{-1}AP$ for Some invertible matrix P. Now

$$\lambda I - B = \lambda I - P^{-1}AP = P^{-1}\lambda IP - P^{-1}AP = P^{-1}(\lambda I - A)P$$

So, $|\lambda I - B| = |P^{-1}(\lambda I - A)P|$

$= |P^{-1}|\lambda I - A|P| = |\lambda I - A|$ as det $|P^{-1}| = (\det |P^{-1}|)$

Thus A and B have the Same Characteristic equation and the Same Characteristic roots.

Scanned with ACE Scanner

We already noted that the Characteristic roots of a real Symmetric matrix are all real, If A is a real Symmetric matrix of order n with Characteristic roots $\lambda_1, \lambda_2, .. \lambda_n$ then there exists a real orthogonal matrix P Such that $P^{-1}AP =$ the diagonal matrix $(\lambda_1, \lambda_2, ... \lambda_n)$ [A real matrix P is Said to be orthogonal if $P^{-1} = P^T$.]

Given a real Symmetric matrix A, find its Characteristic roots $\lambda_1, \lambda_2, ... \lambda_n$. Arrange the roots So that $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$. Then A is Similar to diag $(\lambda_1, \lambda_2, ... \lambda_n)$. To find the matrix P, to each λ, find an eigen Vector x; associates with λ_i. If λ_i is a multiple root of the Characteristic equation $|\lambda I - A| = 0$ (i.e if λ_i is a repeated root) of multiplicity r_i Select eigen Vectors associated with λ Such that they are mutually orthogonal, If $x = (x_1, x_2, ... x_n)$ and $y = (y_1, y_2 ... y_n)$ we Say x and y are Orthogonal if and only if $x_1y_1 + x_2y_2 + ... x_ny_n = 0$).

Now, normalize the eigen Vectors $x_1, x_2, ... x_n$ and obtain $x_1', x_2', ... x_n'$. Then $P = [x_1', x_2', ... x_n']$

Characteristic Root :

Let A be a $n \times n$ Square matrix over a field F and I be the unit matrix of the Same order. The determinent $|A - \lambda I|$ is Called the Characteristic Polynomial of the matrix A. The Equation $\det|A - \lambda I| = 0$ is ~~the~~ Called the Characteristic equation of matrix A. The root of this equation is Called Characteristic roots of matrix A. Characteristic roots are also Called latent roots or eigen values.

Characteristic Vector :

A Scalar λ is a Characteristic root of A if and only if their is a non zero vectors $x \in C^n$ Such that $Ax = \lambda x$. Given a Characteristic root A of λ, the non trivial Solution x which Satisfies $Ax = \lambda x$ are Called Characteristic Vector associative

Find the eigen ~~roots~~ values & eigen vector of the matrix $A=\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

The Characteristic Equation $|A-\lambda I|=0$

$$\left|\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}-\lambda\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right|=0$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix}=0$$

$$8-\lambda((21-9\lambda)-16)+6((-18+6\lambda)-(-8))$$
$$+2(24-(14-2\lambda))$$
$$8-\lambda(5-9\lambda)+6(-10+6\lambda)+2(10+2\lambda)$$
$$40-72\lambda-5\lambda+9\lambda^2-60+36\lambda+20+4\lambda$$
$$9\lambda^2-77\lambda+40\lambda-\cancel{60}+\cancel{60}$$
$$9\lambda^2-37\lambda$$

$$8-\lambda[(7-\lambda)(3-\lambda)-16]+6[-6(3-\lambda)+8]$$
$$+2[(24-(7-\lambda)(2)]=0$$
$$8-\lambda[21-10\lambda+\lambda^2-16]+6[-18+6\lambda+8]+2$$
$$[24-14+2\lambda]=0$$
$$8-\lambda[\lambda^2-10\lambda+5]+6[6\lambda-10]+2[2\lambda+10]=0$$
$$8\lambda^2-80\lambda+40-\lambda^3+10\lambda^2-5\lambda+36\lambda-60+4\lambda+20=0$$
$$-\lambda^3+18\lambda^2-85\lambda+40\lambda+60-60$$
$$-\lambda^3+18\lambda^2-45\lambda=0$$

$$-\lambda^3+18\lambda^2-45\lambda=0$$

$$(-\lambda)\,(\lambda^2-18\lambda+45)=0$$

$$-\lambda=0 \qquad (\lambda-15)(\lambda-3)=0$$

$$\boxed{\lambda=0} \qquad \boxed{\lambda=15} \quad \boxed{\lambda=3}$$

45

$-15 \quad -3$

-18

The roots are $\lambda=0,15,3$

Case(i)

When $\lambda=0$

$$(A-\lambda I)\,x=0$$

$$\left[\begin{pmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{pmatrix} - 0\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\right]\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=0$$

$$\left[\begin{pmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}\right]=0$$

$$8x_1-6x_2+2x_3=0 \rightarrow (1)$$

$$-6x_1+7x_2-4x_3=0 \rightarrow (2)$$

$$2x_1-4x_2+3x_3=0 \rightarrow (3)$$

from (1) & (2)

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & \\ 8 & -6 & 2 & 8\\ -6 & 7 & -4 & -6\\ \cancel{2} & \cancel{-4} & \cancel{3} & \cancel{2}\end{array}$$

$$\frac{x_1}{\underset{10}{24-14}}=\frac{x_2}{\underset{20}{-12+32}}=\frac{x_3}{\underset{20}{56-36}}$$

$$\frac{x_1}{1}=\frac{x_2}{2}=\frac{x_3}{2}$$

$$x=k\begin{bmatrix}1\\2\\2\end{bmatrix}$$

Case (ii)

When $\lambda = 15$

$(A - \lambda I)x = 0$

$$\left[\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{pmatrix}\right] \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\left[\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right] = 0$$

$-7x_1 - 6x_2 + 2x_3 = 0 \rightarrow (1)$

$-6x_1 - 8x_2 - 4x_3 = 0 \rightarrow (2)$

$+2x_1 - 4x_2 - 12x_3 = 0 \rightarrow (3)$

From (2) & (3)

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -6 & -8 & -4 & -6 \\ 2 & -4 & -12 & 2 \end{array}$$

$$\frac{x_1}{96-16} = \frac{x_2}{-8-72} = \frac{x_3}{24+16}$$

$$\frac{x_1}{80} = \frac{x_2}{-80} = \frac{x_3}{40}$$

$$\frac{x_1}{2} = \frac{x_2}{-2} = \frac{x_3}{1}$$

$$\lambda = k\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$$

Case (iii)

When $\lambda = 3$

$(A - \lambda I)x = 0$

$$\left[\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\left[\begin{pmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right] = 0$$

$5x_1 - 6x_2 + 2x_3 = 0 \rightarrow ①$

$-6x_1 + 4x_2 - 4x_3 = 0 \rightarrow ②$

$2x_1 - 4x_2 + 0x_3 = 0 \rightarrow ③$

From ② & ③

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ -6 & 4 & -4 & -6 \\ 2 & -4 & 0 & 2 \end{array}$$

$$\frac{x_1}{0-16} = \frac{x_2}{-8-0} = \frac{x_3}{24-8}$$

$$\frac{x_1}{-16} = \frac{x_2}{-8} = \frac{x_3}{16}$$

$$\frac{x_1}{-2} = \frac{x_2}{-1} = \frac{x_3}{2}$$

$$\lambda = k\begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}$$

?

From ① & ②

$$\frac{x_1}{24-8} = \frac{x_2}{-12+20} = \frac{x_3}{20-36}$$

$$\frac{x_1}{16} = \frac{x_2}{8} = \frac{x_3}{-16}$$

$$\lambda = k\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

H.W

1. Verify whether the System of equation is Consistent & find Solution.

$$x - y + 2z = 3$$
$$x + y + 2z = 6$$
$$3x - y + 6z = 10$$

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 2 \\ 3 & -1 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 6 \\ 10 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$[A, B] = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 1 & 1 & 2 & 6 \\ 3 & -1 & 6 & 10 \end{bmatrix}$$

$$\begin{array}{cccc} 3 & -1 & 6 & 10 \\ -3 & +3 & -6 & -9 \\ \hline 0 & 2 & 0 & 1 \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 2 & 0 & 3 \\ 0 & +2 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$$\rho(A) = 2$$

$$\rho(A, B) = 3$$

$$\rho(A) \neq \rho(A, B)$$

The System of equation is inconsistent & there is no Solution.

---

2. $x + y + z = 6$

$3x - y + 7z = 22$

$6x + 2y + \mu z = \lambda$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 7 \\ 6 & 2 & \mu \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ 22 \\ \lambda \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$[A, B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 3 & -1 & 7 & 22 \\ 6 & 2 & \mu & \lambda \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -4 & 4 & 4 \\ 0 & -4 & \mu-6 & \lambda-36 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 6R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -4 & -4 & 4 \\ 0 & 0 & \mu-10 & \lambda-40 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$\rho(A) = 3$

$\rho(A,B) = 3$

$\rho(A) = \rho(A,B)$

The System of equation is Consistent

(i) InConsistent:

If $\mu = 10$ and $\lambda \neq 40$

$\rho(A) \neq \rho(A,B)$

The System is inConsistent

(ii) Consistent:

If $\mu \neq 10$ and $\lambda \neq 40$

$\rho(A) = 3 \quad \rho(A,B) = 3$

$\rho(A) = \rho(A,B)$

$\therefore$ The System of equation is Consistent.

iii) Consistent and the Solution is unique.

If $\mu \neq 10$ & $\lambda \neq 40$

$\rho(A) = 3$, $\rho(A, B) = 3$

$r = 3$ & $n = 3$

$r = n$

$\therefore$ The System of equation is Consistent & has unique Solution.

iv) Infinite Number of Solution:

If $\mu = 10$ & $\lambda = 40$

$\rho(A) = 2$ $\rho(A, B) = 2$

$r = 2$ & $n = 3$

$r < n$

$\therefore$ The System of equation is Consistent & has infinitly many Solution.

H.W

1. Determinate the Characteristic root of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$.

$-1, \pm\sqrt{3}, 2$

The Characteristic equation $|A - \lambda I| = 0$

$$\left| \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

Scanned with ACE Scanner

3 Find the Characteristic Vector (i) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

(ii) $\begin{bmatrix} \cancel{1} & \cancel{1} & \cancel{3} \\ \cancel{5} & \cancel{2} & \cancel{6} \end{bmatrix}$

The Characteristic equation is $|A - \lambda I| = 0$

$$\left| \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$$

$$\left| \begin{pmatrix} 3-\lambda & 2 \\ 2 & 3-\lambda \end{pmatrix} \right| = 0$$

$$(3-\lambda)(3-\lambda) - 4 = 0$$

$$9 - 3\lambda - 3\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 6\lambda + 5 = 0$$

5

$-1 \quad -5$

-6

$$(\lambda - 1)(\lambda - 5) = 0$$

$$\boxed{\lambda = 1} \quad \boxed{\lambda = 5}$$

The roots are $\lambda = 1, 5$

Case (i)

When $\lambda = 1$

$$(A - \lambda I)x = 0$$

$$\left[\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

Scanned with ACE Scanner

$2x_1 + 2x_2 = 0 \rightarrow ①$

$2x_1 + 2x_2 = 0 \rightarrow ②$

$\frac{x_1}{4-4} = \frac{x_2}{4-4}$

$$\begin{array}{ccc} x_1 & x_2 & \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{array}$$

$\frac{x_1}{0} = \frac{x_2}{0}$

$X = k \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Case (ii)

When $\lambda = 5$

$(A - \lambda I)x = 0$

$\left[\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \right] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

$\begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

$-2x_1 + 2x_2 = 0 \rightarrow ①$

$2x_1 - 2x_2 = 0 \rightarrow ②$

$$\begin{array}{ccc} -2 & 2 & -2 \\ 2 & -2 & 2 \end{array}$$

$\frac{x_1}{4-4} = \frac{x_2}{4-4}$

$\frac{x_1}{0} = \frac{x_2}{0}$

$X = k \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Find the characteristic equation of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ & verify that it is satisfied by A and also find A^{-1}.

The characteristic equation is $|A-\lambda I|=0$

$$\left|\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}\right| = 0$$

$$\left|\begin{bmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{bmatrix}\right| = 0$$

$$2-\lambda\left[(2-\lambda)(2-\lambda)-1\right]+1\left[\overset{(-1)}{\cancel{(2-\lambda)}}(2-\lambda)+1\right]$$
$$+1\left[1-(2-\lambda)\right]=0$$

$$2-\lambda\left[4-2\lambda-2\lambda+\lambda^2-1\right]+1\left[-2+\lambda+1\right]+1$$
$$\left[1-2+\lambda\right]=0$$

$$8-4\lambda-4\lambda+2\lambda^2-2-4\lambda+2\lambda^2+2\lambda^2-\lambda^3+\lambda$$
$$-2+\lambda+\cancel{1}+\lambda-\cancel{1}$$

$$-\lambda^3+6\lambda^2-9\lambda+4=0$$

$$\lambda^3-6\lambda^2+9\lambda-4=0$$

-12λ
$+3\lambda$
-9λ

We have to verify $\lambda = A$ satisfies the equation

$$A^3-6A^2+9A-4I=0$$

$A^2 = A \cdot A$

$$= \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$$A^3 - 6A^2 + 9A - 4I = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$+ 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36+18-4 & -21+30-9-0 & 21-30+9-0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9-0 \\ 21-30+9-0 & -21+30-9-0 & 22-36+18-4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Hence Verified

$$A^3 - 6A^2 + 9A - 4I = 0$$

$$A^3 - 6A^2 + 9A - 4I = \left[\begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix} - 6\begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} + 9\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} - \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix} \right]$$

$\times$ by A^{-1}

$$A^3A^{-1} - 6A^2A^{-1} + 9AA^{-1} - 4IA^{-1} = 0$$

$$A^2 - 6A + 9I - 4A^{-1} = 0$$

$$-4A^{-1} = -A^2 + 6A - 9I$$

$\times$ by $(-)$ on b.s

$$4A^{-1} = A^2 - 6A + 9I$$

$$= \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - 6\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} + \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+12 & -5+6+0 & 6-12+9 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$

4. Find the characteristics equation of $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ & Show that the matrix A Statisfies the Equation.

The Characteristic equation is $|A-\lambda I| = 0$

$$\left| \begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\left| \begin{pmatrix} 1-\lambda & 1 & 3 \\ 5 & 2-\lambda & 6 \\ -2 & -1 & -3-\lambda \end{pmatrix} \right| = 0$$

$$(1-\lambda)\left[(2-\lambda)(-3-\lambda)+6\right] - 1\left[5(-3-\lambda)+12\right] + 3\left[-5-(-2).(2-\lambda)\right] = 0$$

$$(1-\lambda)\left[-\cancel{6}-2\lambda+3\lambda+\lambda^2+\cancel{6}\right] - 1\left[-15-5\lambda+12\right] + 3\left[-5-(-4+2\lambda)\right] = 0$$

$$(1-\lambda)(\lambda^2+\lambda) - 1(-5\lambda-3) + 3(-1-2\lambda) = 0$$

$$\cancel{\lambda^2} + \lambda - \lambda^3 - \cancel{\lambda^2} + 5\lambda + \cancel{3} - \cancel{3} - 6\lambda = 0$$

$$-\lambda^3 - \cancel{5\lambda} + \cancel{5\lambda} = 0$$

$$\lambda^3 = 0$$

We have to verify $\lambda = A$ Satisfies the equation

$$A^3 = 0$$

$A^2 = A.A$

$$= \begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 1+5-6 & 1+2-3 & 3+6-9 \\ 5+10-12 & 5+4-6 & 15+12-18 \\ -2-5+6 & -2-2+3 & -6-6+9 \end{pmatrix}.$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{pmatrix}$$

$A^3 = A^2 . A$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 3+15-18 & 3+6-9 & 9+18-27 \\ -1-5+6 & -1-2+3 & -3-6+9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$A^3 = 0$

$\therefore$ Hence Proved

Cayley Hamilton Theorem:

Every Square matrix Satisfies its own Characteristic Equation.

(i.e) If the Chart Polynomial is

$$\phi(A) = P_0 \lambda^n + P_1 \lambda^{n-1} + P_2 \lambda^{n-2} + \dots P_{n-1} \lambda + P_n$$

Then $\phi(\lambda) = 0$

$$\Rightarrow A^n + P_1 A^{n-1} + P_2 A^{n-2} + \dots P_{n-1} A + P_n = 0$$

Show that the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ Satisfies Cayley hamilton theorem.

The Characteristic equation is $|A - \lambda I| = 0$

$$\left| \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[(3-\lambda)(2-\lambda) - 2] - 2[2-\lambda-1] + 1[2-3+\lambda]$$

$$(2-\lambda)[\lambda^2 - 5\lambda + 6 - 2] - 2[-\lambda+1] + 1(\lambda - 1) = 0$$

$$(2-\lambda)[\lambda^2 - 5\lambda + 4] + 2\lambda - 2 + \lambda - 1 = 0$$

$$2\lambda^2 - 10\lambda + 8 - \lambda^3 + 5\lambda^2 - 4\lambda + 3\lambda - 3 = 0$$

$$-\lambda^3 + 7\lambda^2 - 11\lambda + 5 = 0$$

$$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$$

We have to Verify $\lambda = A$ Satisfies the Equation

$$A^3 - 7A^2 + 11A - 5I = 0$$

$$A^2 = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+1 & 4+6+2 & 2+2+2 \\ 2+3+1 & 2+9+2 & 1+3+2 \\ 2+2+2 & 2+6+4 & 1+2+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 7 \end{bmatrix}$$

$$A^3 = A^2 . A = \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 7 \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12+6 & 14+36+12 & 7+12+12 \\ 12+13+6 & 12+39+12 & 6+13+12 \\ 12+12+7 & 12+36+14 & 6+12+14 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 62 & 31 \\ 31 & 63 & 31 \\ 31 & 62 & 32 \end{bmatrix}$$

$$A^3 - 7A^2 + 11A - 5I = \begin{bmatrix} 32 & 62 & 31 \\ 31 & 63 & 31 \\ 31 & 62 & 32 \end{bmatrix} - 7 \begin{bmatrix} 7 & 12 & 6 \\ 6 & 13 & 6 \\ 6 & 12 & 7 \end{bmatrix}$$

$$+ 11 \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 32-49+22-5 & 62-84+22-0 & 31-42+11-0 \\ 31-42+11-0 & 63-91+33-5 & 31-42+11-0 \\ 31-42+11-0 & 62-84+22-0 & 32-49+22-5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^3 - 7A^2 + 11A - 5I = 0$$

$\therefore$ Hence Proved.

H.W

5. Show that the matrix $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ Statisfy the Cayley Hamilton theorem.

The Characteristic equation is $|A - \lambda I| = 0$

$$\left| \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$$

$$\left| \begin{pmatrix} 1-\lambda & 2 \\ 1 & 1-\lambda \end{pmatrix} \right| = 0$$

$$(1-\lambda)(1-\lambda) - 2 = 0$$

$$1 - \lambda - \lambda + \lambda^2 - 2 = 0$$

$$+\lambda^2 - 2\lambda - 1 = 0$$

$$+\lambda^2 \mp 2\lambda \mp 1 = 0$$

We have to Verify $\lambda = A$ Satisfies the Equation.

$$A^2 \mp 2A \mp I = 0$$

$$A^2 = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+2 & 2+2 \\ 1+1 & 2+1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 3 & 4 \\ 2 & 3 \end{pmatrix}$$

$$A^2 - 2A - 1I = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2-1 & 4-4-0 \\ 2-2-0 & 3-2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 - 2A - 1I = 0$$

$\therefore$ Hence Proved