MARUDHAR KESARI JAIN COLLEGE FOR WOMEN, VANIYAMBADI

PG & RESEARCH DEPARTMENT OF MATHEMATICS

CLASS : I – B.A. ECONOMICS

SUBJECT CODE : 23UEC12

SUBJECT NAME : STATISTICS FOR ECONOMICS –I

SYLLABUS

UNIT- IV

Measures of Dispersion

Absolute and Relative Measures of Dispersion – Range – Quartile Deviation – Mean Deviation – Standard Deviation – Variance - Coefficient of Variation –-Skewness and Kurtosis.

Unit - IV

Measures of Dispersion

The Various Measures of averages give a Single number as the representative of the whole data.

The items are nearer to the mean and is the other they are spread away from the mean.

Two distributions may also have Same median. But the deviations of the observations from the median may be of different type is the two distributions

The measures of dispersion can be classified as the positional measures based on all the observations. The various measures of dispersion.

1) Range

2) Quartile deviation

3) Mean deviation

4) Standard deviation

The first two are positional measures of dispersion and the last two are measures of dispersion based on all the observation.

The measures of dispersion there are two kinds of measures

a) absolute measures of dispersion

b) relative measures of dispersion

Range:

Range is defined to be the difference between the largest and the smallest of the observation.

Range = L - S   L → largest value

S → Smallest Value

co-efficient of Range = $\frac{L-S}{L+S}$

1. The set of observations 13, 25, 36, 22, 18, 45, 21, 26, 30, 22.

$$\text{Range} = L-S$$
$$= 45-13$$
$$= 32$$

$$\text{co-efficient of range} = \frac{L-S}{L+S}$$
$$= \frac{45-13}{45+13}$$
$$= \frac{32}{58} = 0.55$$

Merits and Demerits:

i) It is simple to understand and easy to calculate.

ii) It is unaffected by all other items except the smallest and the largest.

iii) It is affected by the presence of an extremely high or low item.

given point
observations.

Quartile deviation :

In range we consider only the smallest and largest of the observation. It is not a stable measure of dispersion it is very much affected by the extreme value.

The measures of dispersion based on Quartile is used it is called quartile deviation.

The difference between the lowest and this highest of this group is called the inter-quartile range.

$$\text{Inter quartile range} = Q_3 - Q_1$$

$$\text{Quartile deviation} = \frac{Q_3 - Q_1}{2}$$

co-efficient of Quartile deviation = $Q_3 - Q_1$

Merits and Demerits:

i) It is better than range.

ii) It is easy to calculate.

iii) It is useful measures when the extreme classes is a frequenty distribution are not well-defined.

iv) It is not based on all the observation.

v) It is not suitable for mathematical treatment.

vi) It is affected by sampling fluctuations.

Individual series:

1. Find the quartile and the third quartile and also find the Q.D from the given data. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22.

N = No. of items given in question

Q_1 = size of $\left[\frac{N+1}{4}\right]^{th}$ item

$= \text{Size of } \left[\frac{11+1}{4}\right]^{th} \text{ item}$

$= \text{Size of } \left[\frac{12}{4}{}^{th}\right] \text{ item}$

$= 3^{rd} \text{ item}$

$Q_1 = 6$

$Q_3 = \text{Size of } 3\left[\frac{N+1}{4}\right]^{th} \text{ item}$

$= \text{Size of } 3\left[\frac{11+1}{4}\right]^{th} \text{ item}$

$= \text{Size of } 3\left[\frac{12}{4}\right]^{th} \text{ item}$

$= \text{Size of } 3(3)^{rd} \text{ item}$

$= 9^{th} \text{ item}$

$Q_3 = 18$

$$Q.D = \frac{Q_3 - Q_1}{2}$$

$$= \frac{18-6}{2}$$

$$= \frac{12}{2}$$

$Q.D = 6$

co-efficient of Q.D = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$

$$= \frac{18-6}{18+6}$$

$$= \frac{12}{24}$$

$$= 0.5$$

H.W

Find the co-efficient of range from the following data.

Wages	35-45	45-55	55-65	65-75	75-85
No. of. workers	18	22	30	6	4

Soln:-

$\frac{35+45}{2}$ Mid value of the last class (L) = 80

$\frac{75+85}{2}$ Mid value of the first class (S) = 40

$$R = L - S$$

$$R = 80 - 40$$

$$= 40$$

co-efficient of Range = $\frac{L-S}{L+S}$ = 0.33

$$= \frac{80-40}{80+40}$$

$$= \frac{40}{120}$$

3

$$3 \overline{)10}$$ 0.33

9

10

9

1

$$= 0.33$$

continuous series:

2. Find the quartile deviation and the quartile co-efficient of dispersion for the following data.

class	0-5	5-10	10-15	15-20	20-25	25-30
f	3	5	8	12	34	46

class	30-35	35-40	40-45
f	28	14	10

soln:

class	f	cf
0-5	3	3
5-10	5	8
10-15	~~12~~ 8	16
15-20	~~34~~ 12	28

20-25	34	62
25-30	46	108
30-35	28	136
35-40	14	150
40-45	10	160
	N = 160	

$$\frac{N}{4} = \frac{160}{4} = 40$$

The lies between 20-25

$$Q_1 = l + \frac{\frac{N}{4} - cf}{f} \times c$$

$$= 20 + \frac{\frac{160}{4} - 28}{34} \times 5$$

$$= 20 + \frac{12}{34} \times 5$$

$$= 20 + 0.35 \times 5$$

$$= 20 + 1.75$$

$$= 21.75$$

$$Q_3 = L + \left[\frac{3\frac{N}{4} - cf}{f} \right] \times c$$

$$3\left[\frac{N}{4}\right] = 3\left[\frac{160}{4}\right] = 3(40) = 120$$

The lies between 45 - 50

$L =$, $cf =$, $f =$, $c =$

$$Q_3 = \quad + \left[\frac{\quad - \quad}{\quad} \right] \times$$

$$= \quad + \left[\frac{\quad}{\quad} \right] \times 5$$

$$= \quad +$$

$$Q_3 =$$

$$Q.D = \frac{Q_3 - Q_1}{2}$$

$$= \frac{\quad - \quad}{2}$$

$$= \frac{\quad}{2}$$

$$Q.D =$$

co - efficient of Q.D $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

$$= \frac{\quad}{\quad + \quad}$$

=

=

Discrete Series :-

1. calculate the Q.D for the following data find Quartile co-efficient.

Age (in yrs)	20	30	40	50	60	70	80
No. of. members	3	61	132	153	140	51	3

Age	f	cf
20	3	3
30	61	64
40	132	196
50	153	349
60	140	489
70	51	540
80	3	543
	N = 543	

Q_1 = Size of $\left[\frac{N+1}{4}\right]^{th}$ item

= Size of $\left[\frac{543+1}{4}\right]^{th}$ item

= Size of $\left[\frac{544}{4}\right]^{th}$ item

= Size of 136 th item

$Q_1 = 40$

Q_3 = Size of $3\left[\frac{N+1}{4}\right]^{th}$ item

= Size of $3(136)^{th}$ item

= Size of 408 th item

$Q_3 = 60$

$Q.D = \frac{Q_3 - Q_1}{2}$

$= \frac{60-40}{2} = \frac{20}{2} = 10$

Co-efficient of $Q.D = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

$= \frac{60-40}{60+40} = \frac{20}{100}$

$= \frac{1}{5} = 0.2$

continuous series:

$$Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times c$$

$$Q_3 = L + \frac{\frac{3N}{4} - cf}{f} \times c$$

L = Lower limit

Q_3 = Upper limit

Cf = cumulative frequency is preceding of lower quartile.

f = frequency of the lower quartile.

1. calculate the semi-inter quartile range of co-efficient of Q.D

Wages	30-32	32-34	34-36	36-38	38-40	40-42	42-44
Labours	12	18	16	14	12	8	6

Soln:

Wages	f	cf
30 - 32	12	$\boxed{12}$ cf
32 L - 34	$\boxed{18}$ f	30
34 - 36	16	46
36 - 38	14	60 cf
38 L - 40	12 f	72
40 - 42	8	80
42 - 44	6	86
	N = 86	

$\frac{N}{4} = \frac{86}{4} = \underline{21.5}$, $L = 32$, $cf = 12$, $f = 18$

$$Q_1 = L + \frac{\frac{N}{4} - cf}{f} \times c$$

$$= 32 + \frac{21.5 - 12}{18} \times 2$$

$$= 32 + \frac{9.5}{9}$$

$$= 32 + 1.05$$

$$Q_1 = 33.05$$

$3\left[\frac{N}{4}\right] = 3(21.5) = 64.5$

$3\left(\frac{86}{4}\right)$

$L = 38,\ cf = 60,\ f = 12,\ c = 2$

$$Q_3 = L + \frac{\frac{3N}{4} - cf}{f} \times c$$

$$= 38 + \frac{64.5 - 60}{12} \times 2$$

$$= 38 + \frac{4.5}{12} \times 2$$

$$= 38 + 0.75$$

$$= 38.75$$

$$Q.D = \frac{Q_3 - Q_1}{2}$$

$$= \frac{38.75 - 33.05}{2}$$

$$= \frac{5.7}{2}$$

$$Q.D = 2.85$$

co-efficient of $Q.D = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

$$= \frac{38.75 - 33.05}{38.75 + 33.05}$$

$$= \frac{5.7}{71.8}$$

$$= 0.079$$

2. calculate Q.D and its co-efficient of

Months	1	2	3	4	5	6	7	8	9	10	11	12
Monthly earnings	239	250	251	251	257	258	260	261	262	262	273	275

Months (x)	f	cf
1	239	239
2	250	489
3	251	740
4	251	991
5	257	1248
6	258	1506
7	260	1766
8	261	2027
9	262	2289
10	262	2551
11	273	2824
12	275	3099
	N = 3099	

Q_1 = Size of $\left[\frac{N+1}{4}\right]^{th}$ item

= Size of $\left[\frac{3099+1}{4}\right]^{th}$ item

= Size of $\left[\frac{3100}{4}\right]^{th}$ item

= Size of 775^{th} item

$Q_1 = 4$

Q_3 = Size of $3\left[\frac{N+1}{4}\right]^{th}$ item

= Size of $3\left[\frac{3099+1}{4}\right]^{th}$ item

= Size of $3\left[\frac{3100}{4}\right]^{th}$ item

= Size of $3(775)^{th}$ item

= 2325

$Q_3 = 10$

$$Q.D = \frac{Q_3 - Q_1}{2}$$

$$= \frac{10-4}{2} = \frac{6}{2} = 3$$

co-efficient of Q.D $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

$$= \frac{10-4}{10+4}$$

$$= \frac{6}{14}$$

$$= 0.42$$

Mean Deviation (or) Average Deviation.

Definition :

Mean Deviation is the airthmetic mean of the deviation of a Series computed from any measure of central tendency.

(i.e) The Mean, Median or mode all the deviation are taken as position.

(i.e) + and - Signs are ignored

It is denoted by mean deviation

Mean deviation (formula)

Mean deviation ($\bar{x}$) Mean = $\frac{\Sigma |D|}{N}$

(M) Median = $\frac{\Sigma |D|}{N}$

(Z) Mode = $\frac{\Sigma |D|}{N}$

co-efficient of mean deviation (or) relative mean deviation (formula) :-

co-efficient of M.D = $\frac{\text{Mean Deviation}}{\text{Mean (or) Median (or) mode}}$

Individual Series :

Step 1 : calculate the average Mean, Median or mode of the Series

Step 2 : Take the deviations of the items from average and denote those deviations by |D|

Step 3 : compute the total of these deviation. (i.e) $\Sigma |D|$

Step 4 : Divide the total obtained by the number of items

$$M.D = \frac{\Sigma |D|}{N}$$

→ $\Sigma |D|$ = Sum of the deviation

→ N = No. of. items

1. calculate M.D from Mean and Median for the following data 100, 150, 200, 250, 360, 490, 500, 600, 671 also calculate co-efficient of M.D.

$$\text{Mean} = \frac{\Sigma x}{N} \Rightarrow \frac{3321}{9} = 369$$

$$\text{Median} = \text{Size of } \left[\frac{N+1}{2}\right]^{th} \text{item}$$

$$= \text{Size of } \left[\frac{9+1}{2}\right]^{th} \text{item}$$

$$= \text{Size of } \left[\frac{10}{2}\right]^{th} \text{item}$$

$$= \text{Size of } 5^{th} \text{ item}$$

Median = 360

x	$\lvert D\rvert = x - \text{Mean}$ $= x - 369$	$\lvert D\rvert = x - \text{Median}$ $= x - 360$
100	= 100 - 369 269	= 100 - 360 260
150	219	210
200	169	160
250	119	110
360	9	0
490	121	130
500	131	140
600	231	240
671	302	311
$\Sigma x = 3321$	$\Sigma\lvert D\rvert = 1570$	$\Sigma\lvert D\rvert = 1561$

Mean deviation from Mean $= \frac{\Sigma|D|}{N}$

$= \frac{1570}{9}$

$= 174.4$

Mean deviation from Median $= \frac{\Sigma|D|}{N}$

$= \frac{1561}{9}$

$= 173.4$

Co-efficient of Mean deviation $= \frac{\text{Mean deviation}}{\text{Mean}}$

$= \frac{174.4}{369}$

$= 0.47$

Co-efficient of Median deviation $= \frac{\text{Mean deviation}}{\text{Median}}$

$= \frac{176.4}{360}$

$= 0.48$

2. Calculate Mean deviation from Mean and Median for the following data.

7, 4, 10, 9, 15, 12, 7, 9, 7

x	$\|D\| = x - \bar{x}$ $= x - 8.8$	$\|D\| = x - \text{Median}$ $= x - 15$
7	1.8	8
4	4.8	11
10	1.2	5
9	0.2	6
15	6.2	0
12	3.2	3
7	1.8	8
9	0.2	6
7	1.8	8
$\Sigma x = 80$	$\Sigma\|D\| = 21.2$	$\Sigma\|D\| = 55$

$$\text{Mean} = \frac{\Sigma x}{N} \Rightarrow \frac{80}{9} = 8.8$$

$$\text{Median} = \text{Size of } \left[\frac{N+1}{2}\right]^{th} \text{ item}$$

$$= \text{Size of } \left[\frac{9+1}{2}\right]^{th} \text{ item}$$

$$= \text{Size of } \left[\frac{10}{2}\right]^{th} \text{ item}$$

$$= 5^{th} \text{ item}$$

Median = 15

Mean deviation from Mean $= \dfrac{\Sigma |D|}{N}$

$$= \frac{21.2}{9}$$

$$= 2.35$$

Mean deviation from Median $= \dfrac{\Sigma |D|}{N}$

$$= \frac{55}{9}$$

$$= 6.1$$

co-eff of Mean deviation $= \dfrac{\text{Mean deviation}}{\text{Mean}}$

$$= \frac{2.35}{8.8}$$

$$= 0.26$$

co-eff of Median deviation $= \dfrac{\text{Median deviation}}{\text{Median}}$

$$= \frac{6.1}{150}$$

$$= 0.10$$

Discrete Series:

Step 1 : Find out an average (Mean, Median, Mode)

Step 2 : Find out the deviation of the size from the central tendency. ignoring plus (+ or -) or minimize sign and denote then |D|

Step 3 : Multiply the deviation of its size |D| by its respective frequency (f) and find out the total $\Sigma f |D|$

Step 4 : Divide the total by the total frequency Mean deviation $= \frac{\Sigma f |D|}{N}$

1) calculate Mean deviation from mean from the following data

x	2	4	6	8	10
f	1	4	6	4	1

x	f	fx	$\lvert D\rvert = x - \bar{A}$	$f\lvert D\rvert$
2	1	2	$= 2-6$ -4	-4
4	4	16	-2	-8
6 A	6	36	0	0
8	4	32	2	8
10	1	10	4	4
	$\Sigma f = 16$	$\Sigma fx = 96$		$\Sigma f\lvert D\rvert = 24$

Mean $\bar{x} = \dfrac{\Sigma fx}{N}$, $N = \Sigma f = 16$

$= \dfrac{96}{16} = 6$

Mean deviation $= \dfrac{\Sigma f\,\lvert D\rvert}{N}$

$= \dfrac{24}{16}$

$= 1.5$

co-eff of Mean deviation $= \dfrac{\text{Mean deviation}}{\text{Mean}}$

$= \dfrac{1.5}{6} = 0.25$

2) calculate Mean deviation from median from the following Series

x	10	11	12	13	14
f	3	12	18	12	3

x	f	cf	$\lvert D \rvert = x$ - Median $= x - 12$	$f \lvert D \rvert$
10	3	3	2	6
11	12	15	1	12
12	18	33	0	0
13	12	45	1	12
14	3	48	2	6
	$N = 48$			$\Sigma f \lvert D \rvert = 36$

Median = Size of $\left(\frac{N+1}{2}\right)^{th}$ item

= Size of $\left(\frac{48+1}{2}\right)^{th}$ item

= Size of $\left(\frac{49}{2}\right)^{th}$ item

$= 24.5^{th}$ item

Median = 12

Mean deviation $= \frac{\Sigma f|D|}{N} = \frac{36}{48}$

$= 0.75$

co-eff of mean deviation $= \frac{\text{Mean deviation}}{\text{Median}}$

$= \frac{0.75}{12}$

$= 0.0625$

continuous series :

Mean deviation $= \frac{\Sigma f|D|}{N}$

$N = \Sigma f =$ Total frequency

$\Sigma f|D| =$ Sum of the product of frequency

1) calculate the mean deviation from Mean, Median, Mode and also find co-efficient of Mean deviation.

Age in year	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-[illegible]
No. of person	20	25	32	40	42	35	10	8

C.I	f	m	fm	$\lvert D\rvert = x - \text{Mean}$ $= x - 36.50$	$f\lvert D\rvert$
0 - 10	20	5	100	5 - 36.50 = 31.5	630
10 - 20	25	15	375	21.5	537.5
20 - 30	32	25	800	11.5	368
30 - 40	40	35	1400	1.5	60
40 - 50	42	45	1890	8.5	382.5
50 - 60	35	55	1925	18.5	647.5
60 - 70	10	65	650	28.5	285
70 - 80	8	75	600	38.5	308
	N = 212		Σfm = 7740		$\Sigma f\lvert D\rvert$ = 3218.5

Mean :-

$$\text{Mean} = \frac{\Sigma fm}{N} = \frac{7740}{212} \Rightarrow 36.50$$

$$\text{Mean deviation} = \frac{\Sigma f\lvert D\rvert}{N}$$

$$= \frac{3218.5}{212}$$

$$= 151.8$$

$$\text{co-eff of Mean deviation} = \frac{\text{Mean deviation}}{\text{Mean}}$$

$$= \frac{151.8}{36.50} = 4.15$$

Median :-

$$\frac{N}{2} = \frac{212}{2} = 106$$

L = 30 - 40 the lies between

$l = 30,\ Cf = 77,\ f = 40,\ c = 10$

$$\text{Median} = l + \left(\frac{\frac{N}{2} - Cf}{f} \right) \times c$$

$$= 30 + \left(\frac{106 - 77}{40} \right) \times 10$$

$$= 30 + \left(\frac{29}{40} \right) \times 10$$

$$= 30 + 0.725 \times 10$$

$$= 30 + 7.25$$

$$= 37.25$$

C.I	f	m	cf	|D| = x - Median	f |D|
0-10	20	5	20	32.5	650
10-20	25	15	45	22.5	562.5
20-30	32	25	77	12.25	392
30-40	40	35	117	2.25	90
40-50	42	45	159	7.75	325.5
50-60	35	55	194	17.75	621.25

60-70	10	65	204	27.75	277.5
70-80	8	75	212	37.75	301.6
	$\Sigma f =$ 212				$\Sigma f\,\lvert D\rvert =$ 3220.35

Mean deviation $= \dfrac{\Sigma f\,\lvert D\rvert}{N}$

$= \dfrac{3220.35}{212}$

$= 15.19$

co.eff of mean deviation $= \dfrac{\text{Mean deviation}}{\text{Median}}$

$= \dfrac{15.19}{37.25}$

$= 0.40$

Mode :-

$$\text{Mode } (Z) = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times C$$

The lies between 40-50

$L = 40$, $f_1 = 42$, $f_2 = 35$, $f_0 = 40$

$$= 40 + \left(\frac{42-40}{2 \times 42 - 40 - 35}\right) \times 10$$

$$= 40 + \left(\frac{2}{84 - 40 - 35}\right) \times 10$$

$$= 40 + \left(\frac{2}{9}\right) \times 10$$

$$= 40 + 0.222 \times 10$$

$$= 40 + 2.22$$

$$= 42.22$$

Mode grouping table:-

Year	C_1	C_2	C_3	C_4	C_5	C_6
0-10	20	(20+25) 45		(20+25+32) 77		
10-20	25		(25+32) 57		(25+32+40) 97	
20-30	32	(32+40) 72				(32+[illegible]) 114
30-40	40		(40+42) 82	(40+42+35) 117		
40-50	42	(42+35) 77			(42+35+10) 87	
50-60	35		(35+10) 45			(35+10+8) 53
60-70	10	(10+8) 18				
70-80	8					

Analysis table :

Year	C_1	C_2	C_3	C_4	C_5	C_6
0 - 10						
10 - 20					1	
20 - 30					1	1
30 - 40			1	1	1	1
40 - 50	1	1	1	1		1
50 - 60		1		1		
60 - 70						
70 - 80						

Mode = 5

Year	f	m	$\lvert D\rvert = x - \text{mode}$	$f\lvert D\rvert$
0 - 10	20	5	37.22	744.4
10 - 20	25	15	27.22	680.5
20 - 30	32	25	17.22	551.04
30 - 40	40	35	7.22	288.8
40 - 50	42	45	2.78	116.76
50 - 60	35	55	12.78	447.3
60 - 70	10	65	22.78	227.8
70 - 80	8	75	32.78	262.24
	$\Sigma f = 212$			$\Sigma f\lvert D\rvert = 3318.84$

$$\text{Mean deviation} = \frac{\Sigma f|D|}{N}$$

$$= \frac{3318.84}{212}$$

$$= 15.65$$

$$\text{co. eff of Mean deviation} = \frac{\text{Mean deviation}}{\text{Mode}}$$

$$= \frac{15.65}{42.22}$$

$$= 0.3706$$

Merits :

1. It is simple to understand and easy to compute.
2. Mean deviation is a calculated value.
3. It is not much affected by the fluctuations of sampling.
4. It is based on all items of the series and gives weight according to their size.
5. It is less affected by the extreme items
6. It is rigidly defined.

Demerits :-

1. It is non-algebric treatment.
2. It is not a very accurate measure of dispersion.
3. It is not suitable for further mathematical calculation.
4. It is rarely used. It is not as popular as standard deviation.

Standard deviation

Definition :-

It is defined as positive square-root of the arithmetic mean of the square of the deviation of the given observation from their arithmetic mean. The standard deviation is denoted by (σ) sigma.

Individual Series:

(I) Deviation taken from actual mean

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

(II) Deviation taken from assumed mean

$$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$ where $d = x - A$

1) calculate the standard deviation from the following data.

14, 22, 9, 15, 20, 17, 12, 11.

x	x^2
14	196
22	484
9	81
15	225
20	400
17	289
12	144
11	121
$\Sigma x = 120$	$\Sigma x^2 = 1940$

$$\sigma = \sqrt{\frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2}$$

$$= \sqrt{\frac{1940}{8} - \left(\frac{120}{8}\right)^2}$$

$$= \sqrt{242.5 - (15)^2}$$

$$= \sqrt{17.5}$$

$$= 4.18$$

Assumed mean

calculate the standard deviation from the following data.

5, 10, 20, 25, 40, 42, 45, 48, 70, 80

S. No	Marks	$d = x - A$	d^2
1	5	$5 - 40 = -35$	1225
2	10	$10 - 40 = -30$	900
3	20	$20 - 40 = -20$	400
4	25	$25 - 40 = -15$	225
5	40	$40 - 40 = 0$	0
6	42	$42 - 40 = 2$	4
7	45	$45 - 40 = 5$	25
8	48	$48 - 40 = 8$	64
9	70	$70 - 40 = 30$	900
10	80	$80 - 40 = 40$	1600
		$\Sigma d = -15$	$\Sigma d^2 = 5343$

$$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$$= \sqrt{\frac{5343}{10} - \left(\frac{-15}{10}\right)^2}$$

$$= \sqrt{534.3 - (-1.5)^2}$$

$$= \sqrt{534.3 - 2.25}$$

$$= \sqrt{532.05}$$

$$\sigma = 23.06$$

Discrete Series :-

There are three methods for calculating Standard deviation is discrete Series.

a) Actual Mean deviation

$$\sigma = \sqrt{\frac{\Sigma f d^2}{N}} \qquad N = \Sigma f$$

Where $d = x - \bar{x}$

b) Assumed mean method :-

$$\sigma = \sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N}\right)^2}$$

Where $d = x - A$

c) Step-deviation method :

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times c$$

$$\text{Where } d' = \frac{x - A}{c}$$

1) calculate standard deviation from the following data.

Marks	10	20	30	40	50	60
No. of students	8	12	20	10	7	3

a) Actual Method

x	f	fx	$d = x - \bar{x}$ $= x - 30.8$	d^2	fd^2
10	8	80	-20.8	432.64	3461.12
20	12	240	-10.8	116.64	1399.68
30	20	600	-0.8	0.64	12.8
40	10	400	9.2	84.64	846.4
50	7	350	19.2	368.64	2580.48
60	3	180	29.2	852.64	2557.92
	$\Sigma f = 60$	$\Sigma fx = 1850$			$\Sigma d^2 = 10858.4$

$$\text{Mean } \bar{x} = \frac{\Sigma fx}{N}$$

$$= \frac{1850}{60}$$

$$\bar{x} = 30.8$$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N}}$$

$$= \sqrt{\frac{10858.4}{60}}$$

$$= \sqrt{180.97}$$

$$\sigma = 13.45$$

b) Assumed Mean Method :

x	f	$d = x - A$ $= x - 30$	d^2	fd	fd^2
10	8	$10 - 30 = -20$	400	-160	3200
20	12	$20 - 30 = -10$	100	-120	1200
30	20	$30 - 30 = 0$	0	0	0
40	10	$40 - 30 = 10$	100	100	1000
50	7	$50 - 30 = 20$	400	140	2800
60	3	$60 - 30 = 30$	900	90	2700
	$\Sigma f = 60$			$\Sigma fd = 50$	$\Sigma fd^2 = 10900$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \qquad N = \Sigma f$$

$$= \sqrt{\frac{10900}{60} - \left(\frac{50}{60}\right)^2}$$

$$= \sqrt{181.666 - (0.083)^2}$$

$$= \sqrt{181.666 - 0.006889}$$

$$= \sqrt{181.65}$$

$$\sigma = 13.45$$

c) Step-deviation method :

Marks	f	$d' = \frac{x - A}{c}$	d'^2	fd'	fd'^2
10	8	$\frac{10-30}{10} = -2$	4	-16	32
20	12	$\frac{20-30}{10}$ -1	1	-12	12
30	20	$\frac{30-30}{10}$ 0	0	0	0
40	10	$\frac{40-30}{10}$ 1	1	10	10
50	7	2	4	14	28
60	3	3	9	9	27
	$\Sigma f = 60$			$\Sigma fd' = 5$	$\Sigma fd'^2 = 109$

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times c$$

$$= \sqrt{\frac{109}{60} - \left(\frac{5}{60}\right)^2} \times 10$$

$$= \sqrt{1.816 - (0.083)^2} \times 10$$

$$= \sqrt{1.816 - 0.006889} \times 10$$

$$= \sqrt{1.80911} \times 10$$

$$= 1.3450 \times 10$$

$$\sigma = 13.45$$

Continuous Series :

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times c$$

1) Compute the standard deviation from the following data.

x	0-10	10-20	20-30	30-40	40-50	50-60	60-70
f	8	12	17	14	9	7	4

C.I	f	m	$d' = \frac{m-A}{c}$	d'^2	fd'	fd'^2
0 - 10	8	5	$\frac{5-35}{10} = -3$	9	-24	72
10 - 20	12	15	-2	4	-24	48
20 - 30	17	25	-1	1	-17	17
30 - 40	14	35	0	0	0	0
40 - 50	9	45	1	1	9	9
50 - 60	7	55	2	4	14	28
60 - 70	4	65	3	9	12	36
	$\Sigma f = 71$				$\Sigma fd' = -30$	$\Sigma fd'^2 = 210$

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times c$$

$$= \sqrt{\frac{210}{71} - \left(\frac{-30}{71}\right)^2} \times 10$$

$$= \sqrt{2.95 - (-0.42)^2} \times 10$$

$$= \sqrt{2.77} \times 10$$

$$= 1.66 \times 10$$

$$\sigma = 16.6$$

combined Standard deviation:

If a Sample of N_1 item as a mean $\bar{x}_1$ and σ_1 and another Sample of N_2 items as a mean $\bar{x}_2$ and σ_2

We can find out the combined mean and combined Standard deviation by using the formula.

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$\sigma_{12} = \sqrt{\frac{N_1 \sigma_1^2 + N_2 \sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}}$$

Where,

σ_{12} = combined Standard deviation

$d_1 = \bar{x}_1$ - combined mean $(\bar{x}_{12})$

$d_2 = \bar{x}_2$ - combined mean $(\bar{x}_{12})$

Problems :-

1. For a group of 50 male workers, the mean and the standard deviation of their wages are Rs. 63 and Rs. 9 respectively. For a group of 40 female workers those are Rs. 54 and Rs. 6 respectively. Find the standard deviation for the combined group of 90 workers.

characteristics	Groups		combined group
	I Male	II Female	
Size	$N_1 = 50$	$N_2 = 40$	$N_1 + N_2 = 90$
Mean	$\bar{x}_1 = 63$	$\bar{x}_2 = 54$	$\bar{x}_{12} = ?$
S.D	$\sigma_1 = 9$	$\sigma_2 = 6$	$\sigma_{12} = ?$

We know the combined mean

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{50(63) + 40(54)}{50 + 40}$$

$$= \frac{3150 + 2160}{90}$$

$$= \frac{5310}{90}$$

$$\bar{x}_{12} = 59$$

We know that combined standard deviation

$$\sigma_{12} = \sqrt{\frac{N_1 \sigma_1^2 + N_2 \sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}}$$

$$d_1 = \bar{x}_1 - \text{combined mean } (\bar{x}_{12})$$

$$= 63 - 59$$

$$d_1 = 4$$

$$d_2 = \bar{x}_2 - \text{combined mean } (\bar{x}_{12})$$

$$= 54 - 59$$

$$d_2 = -5$$

$$\sigma_{12} = \sqrt{\frac{50(9)^2 + 40(6)^2 + 50(4)^2 + 40(-5)^2}{50 + 40}}$$

$$= \sqrt{\frac{50(81) + 40(36) + 50(16) + 40(25)}{90}}$$

$$= \sqrt{\frac{4050 + 1440 + 800 + 1000}{90}}$$

$$= \sqrt{\frac{7290}{90}}$$

$$= \sqrt{81}$$

$$\sigma_{12} = 9$$

2) A sample of 35 values has mean 80 and S.D 4. A second sample of 65 values has mean 70 and S.D 5. Find the S.D of the combined sample of 100 values.

Given $N_1 = 35$, $N_2 = 65$, $\sigma_1 = 4$, $\bar{x}_1 = 80$,

$\bar{x}_2 = 70$, $\sigma_2 = 5$

We know that combined mean

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$= \frac{35(80) + 65(70)}{35 + 65}$$

$$= \frac{2800 + 4550}{100}$$

$$= \frac{7350}{100}$$

$$\bar{x}_{12} = 73.5$$

We know that combined S.D

$$d_1 = \bar{x}_1 - \text{combined mean } (\bar{x}_{12})$$

$$= 80 - 73.5$$

$$d_1 = 6.5$$

$$d_2 = \bar{x}_2 - \text{combined mean } (\bar{x}_{12})$$

$$= 70 - 73.5$$

$$d_2 = -3.5$$

$$\sigma_{12} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}}$$

$$= \sqrt{\frac{35(4)^2 + 65(5)^2 + 35(6.5)^2 + 65(-3.5)^2}{35 + 65}}$$

$$= \sqrt{\frac{35(16) + 65(25) + 35(42.25) + 65(12.25)}{100}}$$

$$= \sqrt{\frac{560 + 1625 + 1478.75 + 796.25}{100}}$$

$$= \sqrt{\frac{4460}{100}}$$

$$= \sqrt{44.6}$$

$$\sigma_{12} = 6.67$$

3) For a group containing 100 observation the arithmetic mean and S.D are $\overset{\bar{x}_{12}}{\underline{8}}$ and $\overset{\sigma_{12}}{\underline{10.5}}$ for 50 observations selected from these 100 observations the mean and the S.D are $\overset{\bar{x}}{\underline{10}}{}'$ and $\overset{\sigma}{\underline{2}}{}'$ respectively. Find the Arithmetic mean and standard deviation of the other half.

$$N_1 = 50, \quad N_2 = 50$$

$$\bar{x}_1 = 10, \quad \bar{x}_{12} = 8, \quad \bar{x}_2 = ?$$

$$\sigma_1 = 2, \quad \sigma_{12} = 10.5, \quad \sigma_2 = ?$$

We know that combined mean

$$\bar{x}_{12} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$8 = \frac{50(10) + 50(\bar{x}_2)}{50 + 50}$$

$$8 = \frac{500 + \bar{x}_2 50}{100}$$

$$800 = 500 + 50\bar{x}_2$$

$$800 - 500 = 50\bar{x}_2$$

$$300 = 50\bar{x}_2$$

$$\bar{x}_2 = \frac{300}{50}$$

$$\bar{x}_2 = 6$$

We know that combined S.D

$d_1 = \bar{x}_1 -$ combined Mean $(\bar{x}_{12})$

$= 10 - 8$

$d_1 = 2$

$d_2 = \bar{x}_2 -$ combined Mean $(\bar{x}_{12})$

$= 6 - 8$

$d_2 = 2$

$$\sigma_{12} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}}$$

$$10.5 = \sqrt{\frac{50(2)^2 + 50(\sigma_2)^2 + 50(2)^2 + 50(-2)^2}{50+50}}$$

$$10.5 = \sqrt{\frac{50(4) + 50(\sigma_2)^2 + 50(4) + 50(4)}{100}}$$

$$10.5 = \sqrt{\frac{200 + 50\sigma_2^2 + 200 + 200}{100}}$$

$$10.5 = \sqrt{\frac{600 + 50\sigma_2^2}{100}}$$

Squaring on both side

$$(10.5)^2 = \frac{600 + 50\sigma_2^2}{100}$$

$$110.25 = \frac{600 + 50\sigma_2^2}{100}$$

$$11025 = 600 + 50\sigma_2^2$$

$$11025 - 600 = 50\sigma_2^2$$

$$10425 = 50\sigma_2^2$$

$$\sigma_2^2 = \frac{10425}{50}$$

$$\sigma_2^2 = 208.5$$

$$\sigma_2 = 14.43$$

Co-efficient of variation [Relative S.D]

Definition :.

The Measures of dispersion based on standard deviation is defined by $\frac{S.D}{Mean} \times 100$. It is called a co-efficient of variation.

$$C.V = \frac{S.D}{Mean} \times 100$$

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

Individual Series :.

1. Price of a particular commodity is 5 years is 2 cities are given below

Price is city A :	20	22	19	23	16
Price is city B :	10	20	18	12	15

From the above data. Find the city which add were stable price.

Price in city A	$d = x - A$	d^2
20	$20 - 19 = 1$	1
22	$22 - 19 = 3$	9
19	$19 - 19 = 0$	0
23	$23 - 19 = 4$	16
16	$16 - 19 = -3$	9
$\Sigma x = 100$	$\Sigma d = 5$	$\Sigma d^2 = 35$

Mean:

$$\text{Mean} = \frac{\Sigma x}{N}$$

$$= \frac{100}{5}$$

$$\bar{x} = 20$$

Standard deviation:

$$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$\Sigma d^2 = 35$, $\Sigma d = 5$, $N = 5$

$$= \sqrt{\frac{35}{5} - \left(\frac{5}{5}\right)^2}$$

$$= \sqrt{7 - 1}$$

$$= \sqrt{6}$$

$\sigma = 2.4$

co-eff of variation :

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{2.4}{20} \times 100$$

$$= 0.12 \times 100$$

$$CV = 12$$

Price in city B	$d = x - A$	d^2
10	$10 - 18 = -8$	64
20	$20 - 18 = 2$	16
18	$18 - 18 = 0$	0
12	$12 - 18 = -6$	36
15	$15 - 18 = -3$	9
$\Sigma x = 75$	$\Sigma d = -15$	$\Sigma d^2 = 113$

Mean :

$$\text{Mean} = \frac{\Sigma x}{N}$$

$$= \frac{75}{5}$$

$$\bar{x} = 15$$

Standard deviation :-

$$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$$= \sqrt{\frac{113}{5} - \left(\frac{-15}{5}\right)^2}$$

$$= \sqrt{22.26 - (-3)^2}$$

$$= \sqrt{22.26 - 9}$$

$$= \sqrt{13.6}$$

$$\sigma = 3.6$$

co. eff of variation

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{3.6}{15} \times 100$$

$$= 0.24 \times 100$$

$$C.V = 24$$

∴ city B ~~add~~ had more stable prices than city A because the co-efficient of variation is lower in city A.

Discrete Series :-

Goal scored by two teams A and B is a foot ball seasons where as follows

No. of goals scored in match	Team A	Team B
0	27	17
1	9	9
2	8	6
3	5	5
4	4	3

By calculating the co-efficient of variation in each case. Find which team may be consider more consistant

Team A :

x	f	fx	d = x - A	d^2	fd	fd^2
0	27	0	-2	4	-54	108
1	9	9	-1	1	-9	9
2	8	16	0	0	0	0
3	5	15	1	1	5	5
4	4	16	2	4	8	16
	$\Sigma f = 53$	$\Sigma fx = 56$		$\Sigma d^2 = 10$	$\Sigma fd = -50$	$\Sigma fd^2 = 138$

Mean :

$$\text{Mean } (\bar{x}) = \frac{\Sigma fx}{N}$$

$$= \frac{56}{53}$$

$$\bar{x} = 1.05$$

Standard deviation :

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{138}{53} - \left(\frac{-50}{53}\right)^2}$$

$$= \sqrt{2.60 - (-0.9)^2}$$

$$= \sqrt{2.60 - 0.81}$$

$$= \sqrt{1.8}$$

$$\sigma = 1.3$$

co-eff of variation

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{1.3}{1.05} \times 100$$

$$= 1.23 \times 100$$

$$CV = 123$$

Team B :-

x	f	fx	$d = x - A$	d^2	fd	fd^2
0	17	0	-2	4	-34	68
1	9	9	-1	1	-9	9
2	6	12	0	0	0	0
3	5	15	1	1	5	5
4	3	12	2	4	6	12
	$\Sigma f = 40$	$\Sigma fx = 48$		$\Sigma d^2 = 10$	$\Sigma fd = -32$	$\Sigma fd^2 = 94$

Mean :

$$\text{Mean} = \frac{\Sigma f x}{N}$$

$$= \frac{48}{40}$$

$$\bar{x} = 1.2$$

Standard deviation :

$$\sigma = \sqrt{\frac{\Sigma f d^2}{N} - \left(\frac{\Sigma f d}{N}\right)^2}$$

$$= \sqrt{\frac{94}{40} - \left(\frac{-32}{40}\right)^2}$$

$$= \sqrt{2.35 - (-0.8)^2}$$

$$= \sqrt{2.35 - 0.64}$$

$$= \sqrt{1.71}$$

$$\sigma = 1.30$$

co-eff of variation :

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{1.3}{1.2} \times 100$$

$$= 1.08 \times 100$$

$$CV = 108$$

∴ Hence the team A is more consistent then Team B.

continuous Series :

1) calculate the co-efficient of variation for the data

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of men	3	61	132	153	140	51	2

C.I	f	m	$d' = \frac{m-A}{c}$	d'^2	fd'	fd'^2	fm
20-30	3	25	-3	9	-9	27	75
30-40	61	35	-2	4	-122	244	2135
40-50	132	45	-1	1	-132	132	5940
50-60	153	55	0	0	0	0	8415
60-70	140	65	1	1	140	140	9100
70-80	51	75	2	4	102	204	3825
80-90	2	85	3	9	6	18	170
	$\Sigma f = 542$				$\Sigma fd' = -15$	$\Sigma fd'^2 = 765$	Σfm 29660

Mean :

$$\text{Mean } \bar{x} = \frac{\Sigma fm}{\Sigma f}$$

$$= \frac{29660}{542}$$

$$\bar{x} = 54.72$$

Standard deviation :-

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times c$$

$$= \sqrt{\frac{765}{542} - \left(\frac{-15}{542}\right)^2} \times 10$$

$$= \sqrt{1.41 - (-0.02767)^2} \times 10$$

$$= \sqrt{1.41 - 0.0765} \times 10$$

$$= \sqrt{1.0635} \times 10$$

$$= 1.031 \times 10$$

$$\sigma = 10.31$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{10.31}{54.72} \times 100$$

$$= 0.1885 \times 100$$

$$C.V = 18.85$$

Merits :-

1. It is most important and widely used measures of dispersion.

2. It is possible for further algebraic treatment.

3. It is the basis for measuring the co-efficient of correlation sampling and statistical inferences.

4. The standard deviation provides the unit of measurement for the normal distribution.

5. It can be used to calculate the combines standard deviation of two or more groups.

Demerits :-

1. It is not easy to understand and it is difference to calculate.

2. It gives more weight to extreme valu

3. It is affected by the value of every item in the series.

4. It has not found favour with the economists and businessman.

Uses:-

1. S.D is the best measure of dispersion.
2. It is used is co-efficient of correlation.
3. It is widely used is statistics because it possesses most of the characteristics of an ideal measure of dispersion.
4. It is used sampling theory.
5. It is used to study the symmetrical frequency distribution.

comparing between Mean deviation and standard deviation

Mean deviation	Standard deviation
Deviation are calculated from mean, median, mode	Deviation are calculated only from mean
Algebric signs are ignored while calculating M.D	Algebric signs are taken into account

Measures of Skewness:-

Skewness:-

Skewness is a measure to study a statistical distribution. If a distribution is not symmetrical is called skew.

If the frequency curve as a long tail to the right is called skew to the right.

If the frequency curve as a long tail to the left is called skew to the left.

1. Pearson's co-efficient of Skewness:

$$= \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$$

(or)

$$= \frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}}$$

Empirical Relation:

Mean - mode = 3 (Mean - median)

2. Bowley's co-efficient of Skewness:

$$= \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1} \quad \text{(or)} \quad \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

where $Q_1 = l_1 + \frac{N/4 - m_1}{f_1} \times c$

$$Q_2 = l_2 + \frac{N/2 - m_2}{f_2} \times c \quad (\text{or}) \quad l_2 + \frac{2N/4 - m_2}{f_2} \times c$$

$$Q_3 = l_3 + \frac{3N/4 - m_3}{f_3} \times c$$

where l_1 - lower limit of the Q_1 class

m_1 - cummulative frequency of preceeding class

f_1 - frequency of the Q_1 class

C - class interval

N - Total frequency

Q_2 - Second Quartile is called median

	Individual	Discrete	continuous
Mean ($\bar{x}$)	$\bar{x} = \frac{\Sigma x}{N}$ N - Number of observation	$\bar{x} = \frac{\Sigma fx}{N}$ N - Total frequency	$\bar{x} = A \pm \frac{\Sigma fd}{N} \times C$ where, A - Assumed mean N - Total frequency C - class interval $d = \frac{m - A}{C}$
Median (M)	ODD $\frac{n+1}{2}$ EVEN $\frac{n}{2}$ and $\left(\frac{n}{2}+1\right)$	$\bar{x} = \frac{\Sigma fx}{N}$ N - Total frequency	$M = l + \frac{\frac{N}{2} - m}{f} \times$

Mode			$mode = l + \frac{f_1 - f_0}{2f_1 - (f_0+f_2)} \times c$ where, l - lower limit of the model class f_1 - frequency of the model class f_0 - frequency of the premodel class f_2 - frequency of the post model class c - class interval
Standard deviation	$\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$ (or) $\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$	$\sigma = \sqrt{\frac{\Sigma f(x_i - \bar{x})^2}{n}}$ (or) $\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2}$	$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$ $\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$ $\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2}$

1. Find the Pearson's co-efficient of skewness for the following frequency distribution.

Annual Sales (in '000 Rs)	No. of items
0 - 20	20
20 - 40	50
40 - 60	59
60 - 80	30
80 - 100	25
100 - 120	16

Class interval x	Mid value	f	$d = \frac{x-A}{c}$	d^2	fd	$f \times d$ fd^2
0-20	10	20	-2	4	-40	80
20-40	30	50	-1	1	-50	50
40-60	50	59	0	0	0	0
60-80	70	30	1	1	30	30
80-100	90	25	2	4	50	100
100-120	110	16	3	9	48	144
		$N = 200$			$\Sigma fd = 38$	$\Sigma fd^2 = 404$

$$\text{Mean} = A + \frac{\Sigma fd}{N} \times c \qquad A = 50,\ \Sigma fd = 38,\ \Sigma fd^2 = 404$$

$$N = 200$$

$$= 50 + \frac{38}{200} \times 20$$

$$= 50 + 0.19 \times 20$$

$$= 50 + 3.8$$

$$\bar{x} = 53.8$$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

$$l = 40,\ f_1 = 59,\ f_2 = 30,\ f_0 = 50$$

$$= 40 + \frac{59 - 50}{2(59) - 50 - 30}$$

$$= 40 + \frac{9}{118 - 80} \times 20$$

$$= 40 + \frac{9}{38} \times 20$$

$$= 40 + 0.2368 \times 20$$

$$= 40 + 4.73$$

Mode $= \underline{44.73}$

Standard deviation $\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times c$

$$= \sqrt{\frac{404}{200} - \left(\frac{38}{200}\right)^2} \times 20$$

$$= \sqrt{2.02 - (0.19)^2} \times 20$$

$$= \sqrt{2.02 - 0.0361} \times 20$$

$$= \sqrt{1.9839} \times 20$$

$$= 1.408 \times 20$$

$$= \underline{28.16}$$

Pearson's co-efficient $= \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$

$$= \frac{53.8 - 44.73}{28.16}$$

$$= \frac{9.07}{28.16}$$

$$= 0.32$$

2. calculate the Pearson's co-efficient of skewness for the following data.

25, 15, 23, 40, 27, 25, 23, 25, 20

x	$d = x - A$	d^2
25	-2	4
15	-12	144
23	-4	16
40	13	169
27	0	0
25	-2	4
23	-4	16
25	-2	4
20	-7	49
$\Sigma x = 223$	$\Sigma d = -20$	$\Sigma d^2 = 406$

$$\text{Mean} = \frac{\Sigma x}{N}$$

$$= \frac{223}{9}$$

$\bar{x} = 24.7$

Mode $= 25$ 25 occurs three times

$\therefore$ unimodal

Standard deviation $\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$

$= \sqrt{\frac{406}{9} - \left(\frac{-20}{9}\right)^2}$

$= \sqrt{\frac{406}{9} - (-2.22)^2}$

$= \sqrt{45.11 - 4.92}$

$= \sqrt{40.19}$

$= 6.339$

Pearson's co-efficient $= \dfrac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$

$= \dfrac{24.7 - 25}{6.339}$

$= \dfrac{-0.3}{6.339}$

$= -0.04$

H/w sums

calculate the Pearson's co-efficient of skewness for the following data.

7, 4, 10, 9, 15, 4, 12, 7, 9, 7

x	$d = x - A$	d^2
7	-8	64
4	-11	121
10	-5	25
9	-6	36
15	0	0
12	-3	9
7	-8	64
9	-6	36
7	-8	64
$\Sigma x = 80$	$\Sigma d = -55$	$\Sigma d^2 = 419$

$$\text{Mean} = \frac{\Sigma x}{N}$$

$$= \frac{80}{9}$$

$$= 8.888$$

$$\text{Mode} = 7$$

$$\text{Standard deviation } \sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$$

$$= \sqrt{\frac{419}{9} - \left(\frac{-55}{9}\right)^2}$$

$$= \sqrt{46.55 - (-6.11)^2}$$

$$= \sqrt{46.55 - 37.33}$$

$$= \sqrt{9.22}$$

$$= 3.03$$

$$\text{Pearson's co-efficient} = \frac{\text{Mean} - \text{Mode}}{\text{standard deviation}}$$

$$= \frac{8.88 - 7}{3.03}$$

$$= \frac{1.88}{3.03}$$

$$= 0.62$$

Find the Pearson's co-efficient of skewness from the following data.

x	3	4	5	6	7	8	9	10
Size	7	10	14	35	102	136	49	8

x	f	fx	$d = x - A$ $= x - 6$	d^2	fd	fd^2
3	7	21	-3	9	-21	63
4	10	40	-2	4	-20	40
5	14 f_0	70	-1	1	-14	14
6 A	35 f_1	210	0	0	0	0
7	102 f_2	714	1	1	102	102
8	136	1088	2	4	272	544
9	43	387	3	9	129	387
10	8	80	4	16	32	128
	$\Sigma f =$ 355	$\Sigma fx =$ 2610			$\Sigma fd =$ 480	$\Sigma fd^2 =$ 1278

$$\text{Mean} = \frac{\Sigma fx}{N}$$

$$= \frac{2610}{355}$$

$$\bar{x} = 7.352$$

Mode = 3 (inital value)

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

$$= \sqrt{\frac{1278}{355} - \left(\frac{480}{355}\right)^2}$$

$= \sqrt{3.6 - (1.352)^2}$

$= \sqrt{3.6 - 1.8279}$

$= \sqrt{1.7721}$

$\sigma = 1.3312$

Pearson's co-efficient of skewness

$$= \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$$

$$= \frac{7.352 - 3}{1.3312}$$

$$= \frac{4.352}{1.3312}$$

$= 3.269$

Home work Sums

1. Find the Pearson's co-efficient of skewness from the following data

class	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 89
frequency	5	9	14	20	25	15	8	4

C.I	x	m	f	$d = \frac{m-A}{c}$	d^2	fd	fd^2
10 - 19	9.5-19.5	14.5	5	-3	9	-15	45
20 - 29	19.5-29.5	24.5	9	-2	4	-18	36
30 - 39	29.5-39.5	34.5	14 f_0	-1	1	-14	14
40 - 49	39.5-49.5	44.5 A	20 f_1	0	0	0	0
50 - 59	49.5-59.5	54.5	25 f_2	1	1	25	25
60 - 69	59.5-69.5	64.5	15	2	4	30	60
70 - 79	69.5-79.5	74.5	8	3	9	24	72
80 - 89	79.5-89.5	84.5	4	4	16	16	64
			N = 100			Σfd = 48	Σfd^2 = 316

$$\text{Mean} = A \pm \frac{\Sigma fd}{N} \times c$$

$$= 44.5 + \frac{48}{100} \times 10$$

$$= 44.5 + 48$$

$$\bar{x} = 49.5$$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

$$= 39.5 + \frac{20 - 14}{2(20) - 14 - 25}$$

$$= 39.5 + \frac{6}{40-39} \times 10$$

$$= 39.5 + \frac{6}{1} \times 10$$

$$= 39.5 + 60$$

Mode $= 99.5$

Standard deviation $\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2} \times c$

$$= \sqrt{\frac{316}{100} - \left(\frac{48}{100}\right)^2} \times 10$$

$$= \sqrt{3.16 - (0.48)^2} \times 10$$

$$= \sqrt{2.9296} \times 10$$

$$= 1.7116 \times 10$$

$$= 17.116$$

Pearson's co-efficient of skewness

$$= \frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$$

$$= \frac{49.3 - 99.5}{17.116}$$

$$= \frac{-50.2}{17.116}$$

$$= -2.932$$

Bowley's co-efficient of skewness

1. Find out the Bowley's co-efficient of skewness from the following data.

Mid value	21	27	33	39	45	51	57
frequency	18	22	40	50	38	12	4

$\frac{6}{2} = 3$

M.V	C.I	f	cf
21	18 - 24	18	18
27	24 - 30	22	40 m_1
33	l_1 30 - 36	40 f_1	80 m_2
39	l_2 36 - 42	50 f_2	130 m_3
45	l_3 42 - 48	38 f_3	168
51	48 - 54	12	180
57	54 - 60	4	184
		N = 184	

$$B = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$\left(Q_1 = l_1 + \frac{\frac{N}{4} - m_1}{f_1} \times c \right)$$

$$= 30 + \frac{\frac{184}{4} - 40}{40} \times 6$$

$$= 30 + \frac{46 - 40}{40} \times 6$$

$$= 30 + \frac{6}{40} \times 6$$

$$= 30 + 0.15 \times 6$$

$$= 30 + 0.9$$

$$Q_1 = 30.9$$

$$\left(Q_3 = l_3 + \frac{\frac{3N}{4} - m_3}{f_3} \times c \right)$$

$$= 42 + \frac{\frac{3(184)}{4} - 130}{38} \times 6$$

$$= 42 + \frac{\frac{552}{4} - 130}{38} \times 6$$

$$= 42 + \frac{138 - 130}{38} \times 6$$

$$= 42 + \frac{8}{38} \times 6$$

$$= 42 + 0.210 \times 6$$

$$= 42 + 1.26$$

$$Q_3 = 43.26$$

$$\left(Q_2 = l_2 + \frac{\frac{N}{2} - m_2}{f_2} \times c \right)$$

$$= 36 + \frac{\frac{184}{2} - 80}{50} \times 6$$

$$= 36 + \frac{92 - 80}{50} \times 6$$

$$= 36 + \frac{12}{50} \times 6$$

$$= 36 + 0.24 \times 6$$

$$= 36 + 1.44$$

$$Q_2 = 37.44$$

Bowley's co-efficient of skewness

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{43.26 + 30.9 - 2(37.44)}{43.26 - 30.9}$$

$$= \frac{74.16 - 74.88}{12.36}$$

$$= \frac{-0.72}{12.36}$$

$$= -0.058$$

Home work Sums

1. Payments of commission   No. of. Salesman

100 - 120	4
120 - 140	10
140 - 160	16
160 - 180	29
180 - 200	52
200 - 220	80
220 - 240	42
240 - 260	23
260 - 280	17
280 - 300	7

C.I	f	cf
100 - 120	4	4
120 - 140	10	14
140 - 160	16	30
160 - 180	29	59 m_1
l_1 180 - 200	52 f_1	111 m_2
l_2 200 - 220	80 f_2	191 m_3
l_3 220 - 240	42 f_3	233
240 - 260	23	256
260 - 280	17	273
280 - 300	7	280
	N = 280	

$Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \frac{280}{4} = 70$

$Q_2 = \left(\frac{N}{2}\right)^{th}$ item $= \frac{280}{2} = 140$

$Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item $= 3(70) = 210$

$$Q_1 = l_1 + \frac{\frac{N}{4} - m_1}{f_1} \times c$$

$$= 180 + \frac{70 - 59}{52} \times 20$$

$$= 180 + \frac{11}{52} \times 20$$

$$= 180 + 0.2115 \times 20$$

$$= 180 + 4.23$$

$$= 184.23$$

$$Q_2 = l_2 + \frac{\frac{N}{2} - m_2}{f_2} \times c$$

$$= 200 + \frac{140 - 111}{80} \times 20$$

$$= 200 + \frac{29}{80} \times 20$$

$$= 200 + 0.3625 \times 20$$

$$= 200 + 7.25$$

$$Q_2 = 207.25$$

$$Q_3 = l_3 + \frac{3\left(\frac{N}{4}\right) - m_3}{f_3} \times c$$

$$= 220 + \frac{210 - 191}{42} \times 20$$

$$= 220 + \frac{19}{42} \times 20$$

$$= 220 + 0.452 \times 20$$

$$= 220 + 9.04$$

$$= 229.04$$

Bowley's co-efficient of skewness

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{229.04 + 184.23 - 2(207.25)}{229.04 - 184.23}$$

$$= \frac{413.27 - 414.5}{44.81}$$

$$= \frac{-1.23}{44.81}$$

$$= -0.0274$$

2) For a distribution Bowley's co-efficient of skewness is -0.36. lower Quartile is 8.6 and Median is 12.3. What is the Quartile co-efficient of dispersion.

Quartile co-efficient of dispersion $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

Given :-

$$B.k = -0.36$$

$$Q_1 = 8.6$$

$$Q_2 = 12.3$$

Bowley's co-efficient of skewness $= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$

$$-0.36 = \frac{Q_3 + 8.6 - 2(12.3)}{Q_3 - 8.6}$$

$$-0.36(Q_3 - 8.6) = Q_3 + 8.6 - 24.6$$

$$-0.36Q_3 + 3.096 = Q_3 - 16$$

$$3.096 + 16 = Q_3 + 0.36Q_3$$

$$19.096 = Q_3(1 + 0.36)$$

$$19.096 = Q_3(1.36)$$

$$Q_3 = \frac{19.096}{1.36}$$

$$Q_3 = 14.0411$$

Sub Q_3 & Q_1 in eqn ①

co-efficient of dispersion $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

$= \dfrac{14.0411 - 8.6}{14.0411 + 8.6}$

$= \dfrac{5.4411}{22.6411}$

$= 0.2403$

Home work sums :

Find out the Bowley's co-efficient of Skewness from the following data.

Mid value	75	100	125	150	175	200	225	250
Frequency	35	40	48	100	125	80	50	22

M.V	C.I	f	cf
75	62.5 - 87.5	35	35
100	87.5 - 112.5	40	75
125	112.5 - 137.5	48	123 m
150	l_1 137.5 - 162.5	100 f_1	223 m
175	l_2 162.5 - 187.5	125 f_2	348 m
200	l_3 187.5 - 212.5	80 f_3	428
225	212.5 - 237.5	50	478
250	237.5 - 262.5	22	500
		$N = 500$	

$Q_1 = \left(\frac{N}{4}\right)^{th}$ item $= \frac{500}{4} = 125$

$Q_2 = \left(\frac{N}{2}\right)^{th}$ item $= \frac{500}{2} = 250$

$Q_3 = 3\left(\frac{N}{4}\right)^{th}$ item $= 3\left(\frac{500}{4}\right) = 3(125) = 375$

$$Q_1 = l_1 + \frac{\frac{N}{4} - m_1}{f_1} \times c$$

$$= 137.5 + \frac{125 - 123}{100} \times 25$$

$$= 137.5 + \frac{2}{100} \times 25$$

$$= 137.5 + 0.02 \times 25$$

$= 137.5 + 0.5$

$Q_1 = 138$

$$Q_2 = l_2 + \frac{\frac{N}{2} - m_2}{f_2} \times c$$

$$= 162.5 + \frac{250 - 223}{125} \times 25$$

$$= 162.5 + \frac{27}{125} \times 25$$

$$= 162.5 + 0.216 \times 25$$

$$= 162.5 + 5.4$$

$$= 167.9$$

$$Q_3 = l_3 + \frac{3\left(\frac{N}{4}\right) - m_3}{f_3} \times c$$

$$= 187.5 + \frac{3\left(\frac{500}{4}\right) - 348}{80} \times 25$$

$$= 187.5 + \frac{375 - 348}{80} \times 25$$

$$= 187.5 + \frac{27}{80} \times 25$$

$$= 187.5 + 0.3375 \times 25$$

$$= 187.5 + 8.43$$

$Q_3 = 195.93$

Bowley's co-efficient of Skewness

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{195.93 + 138 - 2(167.9)}{195.93 + 138}$$

$$= \frac{333.93 - 335.8}{57.93}$$

$$= \frac{-1.87}{57.93}$$

$$= -0.0322$$

3. In distribution mean = 65, median = 70 and co-eff of Skewness is -0.6

Find 1) Mode

2) co-eff of variation

Given :-

Mean $\bar{x}$ = 65

Median = 70

co-eff of Skewness = -0.6

co-eff of Variation $= \frac{\sigma}{\bar{x}} \times 100$ —— ①

co-eff of Skewness = $\frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}}$

$$S.k = \frac{3(65-70)}{S.D}$$

$$\sigma = \frac{3(65-70)}{S.k}$$

$$\sigma = \frac{3(-5)}{-0.6}$$

$$\sigma = \frac{-15}{-0.6}$$

$$\sigma = 25$$

Sub $\sigma = 25$ in eqn 1

co-eff of variation $= \frac{25}{65} \times 100$

$= 0.3846 \times 100$

$= 38.46$

Mean - Mode $= 3(\text{Mean} - \text{Median})$

$65 - \text{Mode} = 3(65-70)$

$65 - \text{Mode} = 3(-5)$

$65 - \text{Mode} = -15$

$-\text{Mode} = -15 - 65$

$-\text{Mode} = -80$

$\text{Mode} = 80$

4. From a cube distribution the Mean value is Rs. 20 and the median price is Rs. 17 if the co-eff of variation is the 20 percentage. Find the Pearson's co-eff of Skewness.

$$\text{co-eff of variation} = \frac{\sigma}{\bar{x}} \times 100$$

$$\text{co-eff of variation} \times \frac{\bar{x}}{100} = \sigma$$

$$\sigma = \frac{20}{100} \times \frac{20}{100}$$

$$\sigma = \frac{400}{10,000}$$

$$\sigma = 0.04$$

$$\text{Pearson's co-eff of Skewness} = \frac{3(\text{Mean} - \text{Median})}{\text{Standard deviation}}$$

$$= \frac{3(20-17)}{0.04}$$

$$= \frac{3(3)}{0.04}$$

$$= \frac{9}{0.04}$$

$$= 225$$

5. In a distribution sum of two Quaronds is 78.2 and its difference is 14.3 and if its median is 35.7. Find the co-efficient of skewness.

$$Q_3 + Q_1 = 78.2$$

$$Q_3 - Q_1 = 14.3$$

$$Q_2 = 35.7$$

Bowley's co-efficient of skewness

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{78.2 - 2(35.7)}{14.3}$$

$$= \frac{78.2 - 71.4}{14.3}$$

$$= \frac{6.8}{14.3}$$

$$= 0.4755$$

Kurtosis Based on Moments:

Definition:

Kurtosis is a measure of platness or Peakness of a distribution.

Types of Kurtosis:

Mesokurtic:

The normal curve (Bell Shaped curve) is called Mesokurtic.

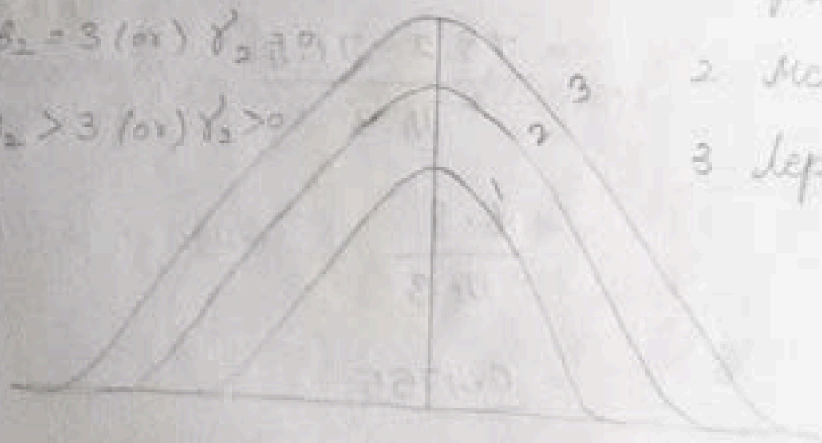
Platykurtic:

The curve which is more platetopped that the normal curve is called platykurtic.

Leptokurtic:

The curve which is more peaked than the normal curve is called leptokurtic

1. $\beta_2 < 3$ (or) $\gamma_2 < 0$
2. $\beta_2 = 3$ (or) $\gamma_2 = 0$
3. $\beta_2 > 3$ (or) $\gamma_2 > 0$



1 plat...
2 Mesoku...
3 lepto...

Measures of kurtosis:

Kurtosis is measured by the co-efficient

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \text{ (or) } \gamma_2 = \frac{\mu_4}{\mu_2^2} - 3$$

For the normal distribution $\gamma_2 = 0$ & $\beta_2 = 3$

If $\beta_2 = 3$ (or) $\gamma_2 = 0$ then the curve is called Mesokurtic.

If $\beta_2 > 3$ (or) $\gamma_2 > 0$ then the curve is called leptokurtic.

If $\beta_2 < 3$ (or) $\gamma_2 < 0$ then the curve is called platykurtic.

μ_2 & μ_4 are the second and fourth central moments.

The measure of skewness based on moments is given by $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

Measure of skewness $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

Measure of kurtosis $\beta_2 = \frac{\mu_4}{\mu_2^2}$

central moments $= \mu_r'$

A = -0.129

B = -0.251

1) The first four central moments of a distribution are 0, 2.5, 0.7, 18.75. Test the skewness & kurtosis of a distribution

The co-eff of skewness is $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

Given:

$\mu_1' = 0$, $\mu_2' = 2.5$, $\mu_3' = 0.7$, $\mu_4' = 8.75$

$\therefore \beta_1 = \frac{(0.7)^2}{(2.5)^3}$

$\mu_1 = \mu_1'$

$\mu_2 = \mu_2' - (\mu_1')^2$

$= 2.5 - 0^2$

$= 2.5$

$= \frac{0.49}{15.625}$

$\beta_1 = 0.031$

β_1 is postive the distribution is positively skewed.

$\therefore$ The co-eff of kurtosis $\beta_2 = \frac{\mu_4}{\mu_2^2}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= 0.7 - 3(2.5)(0) + 2(0)^3$

$= 0.7$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= (8.75) - 4(0.7)(0) + 6(2.5)(0) - 3(0)$

$= 8.75$

$= \frac{8.75}{(2.5)^2}$

$= \frac{8.75}{6.25}$

$\beta_2 = 1.4$

2) The first four moments of the distribution about the value 5 are 2, 20, 40, 150. Find the measure of kurtosis & command on the distribution.

$$\mu_1' = 2, \quad \mu_2' = 20, \quad \mu_3' = 40, \quad \mu_4' = 150$$

$$\left(\mu_2 = \mu_2' - (\mu_1')^2\right)$$

$$= 20 - (2)^2$$

$$= 20 - 4$$

$$= 16$$

$$\left(\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3\right)$$

$$= 40 - 3(20)(2) + 2(2)^3$$

$$= 40 - 3(40) + 2(8)$$

$$= 56 - 120$$

$$= 56 - 120$$

$$\mu_3 = -64$$

$$\left(\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4\right)$$

$$= 150 - 4(40)(2) + 6(20)(2)^2 - 3(2)^4$$

$$= 150 - 4(80) + 6(20)(4) - 3(16)$$

$$= 150 - 320 + 6(80) - 48$$

$$= 150 - 320 + 480 - 48$$

$$= 262$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$= \frac{(-64)^2}{(16)^3}$$

$$= \frac{4096}{4096}$$

$$\beta_1 = 1$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{262}{(16)^2}$$

$$= \frac{262}{256}$$

$$\beta_2 = 1.023$$

Home work Sums

Find the quartile co-eff of skewness of the Two groups below which group is more skewness

Mark	A	B
55 - 58	12	20
58 - 61	17	22
61 - 64	23	25
64 - 67	18	13

Group A

C.I	f	cf
55 - 58	12	12
l_1 58 - 61	17 f_1	29
l_2 61 - 64	23 f_2	52
l_3 64 - 67	18 f_3	70

$$Q_1 = \left(\frac{N}{4}\right)^{th} \text{term}$$

$$= \frac{70}{4}$$

$$Q_1 = 17.5$$

$$Q_2 = \left(\frac{N}{2}\right)^{th} \text{term}$$

$$= \frac{70}{2}$$

$$Q_2 = 35$$

$$Q_3 = 52.5$$

Bowley's co-efficient of skewness

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$Q_1 = \frac{l_1 + \frac{N}{4} - m_1 \times c}{f_1}$$

$$= 58 + \frac{17.5 - 12}{17} \times 3$$

$$= 58 + \frac{5.5}{17} \times 3$$

$$= 58 + 0.323 \times 3$$

$$= 58 + 0.969$$

$$= 58.969$$

$SK_B = 0.512$

$SK_A = -0.048$

$$Q_2 = l_2 + \frac{\frac{N}{2} - m_2}{f_2} \times c$$

$$= 61 + \frac{35 - 29}{23} \times 3$$

$$= 61 + \frac{6}{23} \times 3$$

$$= 61 + 0.260 \times 3$$

$$= 61 + 0.78$$

$$= 61.78$$

$$Q_3 = l_3 + \frac{3\left(\frac{N}{4}\right) - m_3}{f_3} \times c$$

$$= 64 + \frac{52.5 - 52}{18} \times 3$$

$$= 64 + \frac{0.5}{18} \times 3$$

$$= 0.027 \times 3 + 64$$

$$= 64.081$$

Bowley's co-efficient of skewness

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{64.081 + 58.969 - 2(61.78)}{64.081 - 8.969}$$

$$= \frac{123.05 - 123.56}{5.112}$$

$$= \frac{-0.51}{5.112}$$

$$= -0.0997$$

Group B

C.I	f	cf
55 - 58	20	20
58 - 61	22	42
61 - 64	25	67
64 - 67	13	80

$$Q_1 = \left(\frac{N}{4}\right)^{th} \text{ term}$$

$$= \frac{80}{4}$$

$$Q_1 = 20$$

$$Q_2 = \left(\frac{80}{2}\right)^{th} \text{ term}$$

$$= 40$$

$$Q_3 = 3(20) = 60$$

$$Q_1 = l_1 + \frac{\frac{N}{4} - m_1}{f_1} \times c$$

$$= 58 + \frac{20-20}{22} \times 3$$

$$Q_1 = 58$$

$$Q_2 = l_2 + \frac{\frac{N}{2} - m_2}{f_2} \times c$$

$$= 61 + \frac{40-42}{25} \times 3$$

$$= 61 + (-0.08) \times 3$$

$$= 61 - 0.24$$

$$= 60.76$$

$$Q_3 = l_3 + \frac{3\left(\frac{N}{4}\right) - m_3}{f_3} \times c$$

$$= 61 + \frac{60-67}{25} \times 3$$

$$= 61 + \left(\frac{-7}{25}\right) \times 3$$

$$= 61 + (-0.28) \times 3$$

$$= 61 - 0.84$$

$$= 60.16$$

Bowley's co-efficient of skewness

$$= \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

$$= \frac{60.16 + 58 - 2(60.76)}{60.16 - 58}$$

$$= \frac{118.16 - 121.52}{2.16}$$

$$= \frac{-3.36}{2.16}$$

$$= -1.55$$