MARUDHAR KESARI JAIN COLLEGE FOR WOMEN,VANIYAMBADI
PG & RESEARCH DEPARTMENT OF MATHEMATICS

SUBJECT NAME: NUMERICAL METHODS I

CLASS : 1 B.Sc CS

CODE: 23UECS12A

SYLLABUS:

Unit-I Curve Fitting**-** Principle of Least square Fitting of straight line $Y= ax+b$, parabola = $Y=ax^2+bx+c$, exponential curves of forms $Y=ax^b$,$Y=ae^{bx}$,$Y=ab^x$

UNIT - I

NUMERIAL METHOD (curve fitting principle of least square)

curve fitting

The procedure of evaluating unknown constant with the help of given data is known as curve fitting.

Least square method :

The most frequently used method to obtain the closes fit to the given data containing various error of measurement as been discussed the method is known as Least square method

1. Using the method of least square find the best fitting line to the given data

x	1	2	3	4	5
y	1	3	5	6	5

Assume that the line that best fits to given data is $y = a + bx$

$$\Sigma y = an + b\Sigma x \longrightarrow (1)$$

$$\boxed{\Sigma xy = a\Sigma x + b\Sigma x^2} \longrightarrow (2)$$

$\boxed{n = 5}$ Given Data

x	y	xy	x^2
1	1	1	1
2	3	6	4
3	5	15	9
4	6	24	16
5	5	25	25

$\Sigma x = 15$, $\Sigma y = 20$, $\Sigma xy = 71$, $\Sigma x^2 = 55$

$20 = a5 + 15b \rightarrow$ A

$71 = a15 + 55b \rightarrow$ B

mul 3 by equ A

$60 = 15a + 45b$

$71 = 15a + 55b$

$(-) \quad (-) \quad (-)$

$-11 = -10b$

$b = \frac{-11}{-10}$

$b = \frac{11}{10}$

$\boxed{b = 1.1}$ sub in A

$20 = a5 + 15(1.1)$

$20 = a5 + 16.5$

$20 - 16.5 = a5$

$\frac{3.5}{5} = a$

$\boxed{a = 0.7}$ sub in formula

$y = a + bx$

$\boxed{y = 0.7 + 1.1(x)}$

2. Fit a parabola $y = a + bx + cx^2$ in the least square sense for the following data and estimate when $x = 6$

H.W

x	1	2	3	4	5
y	10	12	13	16	19

$$y = a + bx + cx^2$$

$$\Sigma y = an + b\Sigma x + c\Sigma x^2 \rightarrow (1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \rightarrow (2)$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \rightarrow (3)$$

x	y	x^2	x^3	x^4	xy	x^2y
1	10	1	1	1	10	10
2	12	4	8	16	24	48
3	13	9	27	81	39	117
4	16	16	64	256	64	256
5	19	25	125	625	95	475
$\Sigma x = 15$	$\Sigma y = 70$	$\Sigma x^2 = 55$	$\Sigma x^3 = 225$	$\Sigma x^4 = 979$	$\Sigma xy = 232$	$\Sigma x^2y = 906$

$$70 = 5a + 15b + 55c$$

$$232 = 15a + 55b + 225c$$

$$906 = 55a + 225b + 979c$$

$$D = \begin{vmatrix} 5 & 15 & 55 \\ 15 & 55 & 225 \\ 55 & 225 & 979 \end{vmatrix} = 5(55 \times 979 - 225 \times 225) - 15(15 \times 979 - 55 \times 225) + 55(15 \times 225 - 55 \times 55)$$

$$= 700$$

$$D_1 = \begin{vmatrix} 70 & 15 & 55 \\ 232 & 55 & 225 \\ 906 & 225 & 979 \end{vmatrix} = 70(55\times979 - 225\times225) - 15(232\times979 - 225\times906) + 55(232\times225 - 55\times906)$$

$$= 6580$$

$$D_2 = \begin{vmatrix} 5 & 70 & 55 \\ 15 & 232 & 225 \\ 55 & 906 & 979 \end{vmatrix} = 5(232\times979 - 225\times906) - 70(15\times979 - 225\times55) + 55(15\times906 - 232\times55)$$

$$= 340$$

$$D_3 = \begin{vmatrix} 5 & 15 & 70 \\ 15 & 55 & 232 \\ 55 & 225 & 906 \end{vmatrix} = 5(55\times906 - 232\times225) - 15(15\times906 - 232\times55) + 70(15\times225 - 55\times55)$$

$$= 200$$

$$a = \frac{D_1}{D} = \frac{6580}{700} \qquad a = 9.4$$

$$b = \frac{D2}{D} = \frac{340}{700} \qquad b = 0.4857$$

$$c = \frac{D3}{D} = \frac{200}{700} \qquad c = 0.2857$$

$$y = a + bx + cx^2$$

$$y = 9.4 + 0.4857x + 0.2857x^2$$

$x = 6$

$$y = 9.4 + 0.4857(6) + 0.2857(6)^2$$

$$y = 22.5994$$

Find a and b $y = ae^{bx}$ fits the data

x	1	2	3	4
y	7	11	17	27

$\log AB = \log A + \log B$

$\boxed{e^0 = 1}$

$\boxed{\log e^e = 1}$

$$y = ae^{bx}$$

Taking log on both sides

$$\log e^{y} = \log e^{(ae^{bx})}$$

$$= \log e^{a} + \log e^{ebx}$$

$$= \log e^{a} + bx(1)$$

$$\log e^{y} = \log e^{a} + bx$$

$$\therefore Y = \log e^{y} \text{ and } A = \log e^{a}$$

$$Y = a + bx$$

$$\Sigma y = Ana + bx^2$$

$$\Sigma xy = \Sigma Ax + \Sigma bx^2$$

x	y	$Y = \log e^{y}$	xY	x^2
1	7	1.9459	1.9459	1
2	11	2.3979	4.7958	4
3	17	2.8332	8.4996	9
4	27	3.2758	13.1832	16
$\Sigma x =$ 10	$\Sigma y =$ 62	$\Sigma Y =$ 10.4528	$\Sigma xY =$ 28.4245	$\Sigma x^2 =$ 30

$$28.4245 = 10A + 30b \rightarrow (2)$$

$$10.4528 = 4A + 10b \rightarrow (1)$$

$(1)\times 3$

$$31.3584 = 12A + 30b$$

$$28.4245 = 10A + 30b$$

$$2.9339 = 2A$$

$$A = \frac{2.9339}{2}$$

$$A = 1.46695$$

From equ (1)

$$10.4528 = 4(1.46695) + 10b$$

$$10.4528 - 4(1.46695) = 10b$$

$$b = 0.45852$$

$$A = \log_e a$$

$$a = e^A = e^{1.4970}$$

$$a = 4.4683$$

$$y = ae^{bx}$$

$$y = 4.4683\, e^{0.4585(x)}$$

Fit a straight line to the following data.

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

$$\Sigma y = an + b\Sigma x \rightarrow ①$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow ②$$

x	y	x^2	xy
0	1	0	0
1	1.8	1	1.8
2	3.3	4	6.6
3	4.5	9	13.5
4	6.3	16	25.2
$\Sigma x =$ 10	$\Sigma y =$ 16.9	$\Sigma x^2 =$ 30	$\Sigma xy =$ 47.1

$$16.9 = 5a + 10b \rightarrow Ⓐ$$
$$47.1 = 10a + 30b \rightarrow Ⓑ$$

Multi 2 into equ Ⓐ

$$33.8 = 10a + 20b$$
$$47.1 = 10a + 30b$$
$$(-) \quad (-) \quad (-)$$

$$-13.3 = -10b$$

$$b = \frac{13.3}{10} \qquad \boxed{b = 1.33}$$

Sub into equ (A)

$16.9 = 5a + 10b$

$16.9 = 5a + 10(1.33)$

$16.9 - 13.3 = 5a$

$3.6 = 5a$

$a = \frac{3.6}{5}$

$a = 0.72$

Fit a straight line to the following data

x	1	2	3	4	5	6
y	1200	900	600	200	110	50

x	y	x^2	xy
1	1200	1	1200
2	900	4	1800
3	600	9	1800
4	200	16	800
5	110	25	550
6	50	36	300
$\Sigma x =$ 21	$\Sigma y =$ 3060	$\Sigma x^2 =$ 91	$\Sigma xy =$ 6450

$\Sigma y = an + b\Sigma x \rightarrow (1)$

$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow (2)$

$3060 = 6a + 21b \rightarrow (A)$

$6450 = 21a + 91b \rightarrow (B)$

multiple A into (7)

multiple B into (2)

$21420 = 42a + 147b$

$12900 = 42a + 182b$

$8520 = -35b$

$b = -\frac{8520}{35}$

$\boxed{b = -243.4}$

sub in equ (A)

$3060 = 6a + 21b$

$3060 = 6a + 21(-243.4)$

$3060 = 6a - 5111.82$

$3060 + 5111.82 = 6a$

$8171.82 = 6a$

$\frac{8171.82}{6} = a$

$\boxed{a = 1361.97}$

Find the curve of best fit of the type $y = ae^{bx}$

x	1	5	7	9	12
y	10	15	12	15	21

$$y = ae^{bx}$$

$$\log y = \log a + \log e^{bx}$$

$$\log y = \log a + bx \log e$$

$$Y = A + Bx$$

$$Y = \log y \qquad A = \log A$$

$$Bx = Bx \log e$$

$$B = \log e$$

$$\Sigma y = an + b\Sigma x \rightarrow \text{(A)}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow \text{(B)}$$

x	$\log y$	x^2	xy (log)
1	10	1	10
5	15	25	75
7	12	49	84
9	15	81	135
12	21	144	252
$\Sigma x =$ 34	$\Sigma y =$	$\Sigma x^2 =$	$\Sigma xy =$

x	y	$\log y$	x^2	$\log xy$
1	10	1	1	1
5	15	1.1761	25	5.8805
7	12	1.0792	49	7.5544
9	15	1.1761	81	10.5858
12	21	1.3222	144	15.8664
$\Sigma x =$ 34		$\Sigma y =$ 5.7536	$\Sigma x^2 =$ 300	$\Sigma xy =$ 40.8871

$$\Sigma y = an + b\Sigma x \rightarrow (1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow (2)$$

$$5.7536 = 5a + 34b \rightarrow (A)$$

$$40.8871 = 34a + 300b \rightarrow (B)$$

mul 34 in equ (A)

mul 5 in equ (B)

$$195.6224 = 170a + 1156b$$

$$204.4355 = 170a + 1500b$$

$$-8.8131 = -344b$$

$$\frac{8.8131}{344} = b$$

$$b = 0.02561$$

Sub equ in (A)

$$5.7536 = 5a + 34b$$

$$5.7536 = 5a + 34(0.02561)$$

$$5.7536 = 5a + 0.87074$$

$5.7536 - 0.87074 = 5a$

$$\frac{4.88286}{5} = a$$

$$a = 0.9766$$

$$b = \frac{B}{\log_{10} e} = \frac{0.02561}{\log_{10} e}$$

$$=$$

Find the curve of best fit of the type

$y = ax + b$

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

$$\Sigma y = an + b\Sigma x \rightarrow ①$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow ②$$

x	y	x^2	xy
1	1	1	1
3	2	9	6
4	4	16	16
6	4	36	24
8	5	64	40
9	7	81	63
11	8	121	88
14	9	196	126
$\Sigma x =$ 56	$\Sigma y =$ 40	$\Sigma x^2 =$ 524	$\Sigma xy =$ 364

$$40 = 8a + 56b \rightarrow Ⓐ$$

$$364 = 56a + 524b \rightarrow Ⓑ$$

Multiple 7 into equ Ⓐ

$$280 = 56a + 392b$$

$$\begin{array}{rcl} 364 & = & 56a + 524b \\ (-) & & (-) \quad (-) \\ \hline -84 & = & -132b \end{array}$$

$$\frac{84}{132} = b$$

$$\boxed{b = 0.64}$$

sub in equ (A)

$$40 = 8a + 56b$$
$$40 = 8a + 56(0.64)$$
$$40 = 8a + 35.84$$
$$40 - 35.84 = 8a$$
$$4.16 = 8a$$
$$a = \frac{4.16}{8}$$
$$\boxed{a = 0.52}$$

Find the curve of best fit of the type $y = ax + b$ when $x = 30$.

x	5	10	15	20	25
y	16	19	23	26	30

x	y	x^2	xy
5	16	25	80
10	19	100	190
15	23	225	345
20	26	400	520
25	30	625	750
$\Sigma x =$ 75	$\Sigma y =$ 114	$\Sigma x^2 =$ 1375	$\Sigma xy =$ 1885

$$\Sigma y = an + b\Sigma x \rightarrow ①$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow ②$$

$$114 = 5a + 75b \rightarrow Ⓐ$$

$$1885 = 75a + 1375b \rightarrow Ⓑ$$

mul 15 into equ Ⓐ

$$1710 = 75a + 1125b$$

$$\underline{\underset{(-)}{1885} = \underset{(-)}{75a} + \underset{(-)}{1375b}}$$

$$\underline{-175 = -250b}$$

$$\cancel{+}175 = \cancel{+}250b$$

$$b = 0.7$$

sub in equ Ⓐ

$$114 = 5a + 75b$$

$$114 = 5a + 75(0.7)$$

$$114 = 5a + 52.5$$

$$114 - 52.5 = 5a$$

$$61.5 = 5a$$

$$\frac{61.5}{5} = a$$

$$a = 12.3$$

$$y = ax + b$$

$$y = 12.3(30) + 0.7$$

$$y = 369 + 0.7$$

$$y = 369.7$$

Using the method of least squares, find the best fitting line to the given data.

x	1	2	3	4	5
y	1	3	5	6	5

$$\Sigma y = an + b\Sigma x \rightarrow (1)$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow (2)$$

x	y	xy	x^2
1	1	1	1
2	3	6	4
3	5	15	9
4	6	24	16
5	5	25	25
$\Sigma x =$ 15	$\Sigma y =$ 20	$\Sigma xy =$ 71	$\Sigma x^2 =$ 55

$$20 = 5a + 15b \rightarrow (A)$$
$$71 = 15a + 55b \rightarrow (B)$$

mul 3 into equ (A)

$$60 = \cancel{15a} + 45b$$
$$\underset{(-)}{71} = \underset{(-)}{\cancel{15a}} + \underset{(-)}{55b}$$

$$-11 = -10b$$

$$b = \frac{11}{10}$$

$$\boxed{b = 1.1}$$

sub in equ A

$20 = 5a + 15b$

$20 = 5a + 15(1.1)$

$20 = 5a + 16.5$

$20 - 16.5 = 5a$

$\frac{3.5}{5} = a$

$\boxed{a = 0.7}$

$y = a + bx$

$y = 0.7 + 1.1x$

Fit a curve of the form $y = ae^{bx}$ for the data

x	0	2	4
y	8.12	10	31.82

$y = ae^{bx}$

Taking log on both side

$\log y_e = \log a_e + bx \log_e e$

$Y = A + bx$

$\Sigma y = an + b\Sigma x \rightarrow (1)$

$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow (2)$

x	y	$Y = \log_e y$	xy	x^2
0	8.12	2.0943	0	0
2	10	2.3026	4.6052	4
4	31.92	3.4601	13.8404	16
$\Sigma x =$ 6		$\Sigma y =$ 7.8570	$\Sigma xy =$ 18.4456	$\Sigma x^2 =$ 20

$$7.8570 = 3a + 6b \longrightarrow (A)$$
$$18.4456 = 6a + 20b \longrightarrow (B)$$

mul 2 into equ (A)

$$15.714 = 6a + 12b$$
$$18.4456 = 6a + 20b$$
$$(-) \quad (-) \quad (-)$$

$$-2.7316 = -8b$$

$$\frac{2.7316}{8} = b$$

$$\boxed{b = 0.34145}$$

Sub in equ (A)

$$7.8570 = 3a + 6b$$
$$7.8570 = 3a + 6(0.34145)$$
$$7.8570 = 3a + 2.0487$$
$$7.8570 - 2.0487 = 3a$$
$$5.8083 = 3a$$
$$\frac{5.8083}{3} = a$$
$$a = 1.9361$$

$A \log_e a \to a = e^{a}$

$= e^{1.9361}$

$a = 6.9317$

$y = ae^{bx}$

$y = (6.9317)e^{0.3415x}$

Fit an exprotial curve of the form $y = ae^{bx}$ by the method of least square for the following table

No. of petal	5	6	7	8	9	10
No. of flowers	133	55	23	7	2	2

$y = ae^{bx}$

$\log y = \log_e a + bx \log_e e$

$Y = a + bx$

$\Sigma y = an + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

x	y	$y = \log y$	xy	x^2
5	133	4.8903	24.4515	25
6	55	4.0073	24.0438	36
7	23	3.1355	21.9485	49
8	7	1.9459	15.5672	64
9	2	0.6931	6.2379	81
10	2	0.6931	6.9310	100
$\Sigma x =$ 45	~~Σy~~	$\Sigma y =$ 15.3652	$\Sigma xy =$ 99.1799	$\Sigma x^2 =$ 355

$15.3652 = 6a + 45b \rightarrow (A)$

$99.1799 = 45a + 355b \rightarrow (B)$

mul 15 into equ (A) mul 2 into equ (B)

$230.478 = 90a + 675b$

$198.3598 = 90a + 710b$

$(-) \quad (-) \quad (-)$

$32.1182 = -35b$

$-\frac{32.1182}{35} = b$

$b = -0.9177$

sub in equ (A)

$15.3652 = 6a + 45b$

$15.3652 = 6a + 45(-0.9177)$

$15.3652 = 6a - 41.2965$

$15.3652 + 41.2965 = 6a$

$\frac{56.6617}{6} = a$

$a = 9.4436$

$A = \log e^{a} \Rightarrow a = e^{A} = e^{9.4436}$

$a = 12627.09$

$y = ae^{b}$

$y = (12627.09)e^{-0.9177}$

Fit a best fitting curve in the form $y = ax^b$ for the following data

x	1	2	3	4	5	6
y	2.98	4.76	5.4	6.1	6.8	7.5

calculate the value of y when $x = 3.5$.

$$y = ax^b$$

Applying $\log_e$ on both side

$$\log_e y = \log_e [ax^b]$$

$$\log y = \log a + \log x^b$$

$$\log y = \log a + b \log x$$

$$Y = A + bX$$

$$\Sigma y = an + b\Sigma x \rightarrow (1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow (2)$$

x	y	$X = \log x$	$Y = \log y$	X^2	XY
1	2.98	0	1.0919	0	0
2	4.76	0.6931	1.4493	0.4805	1.0045
3	5.4	1.0986	1.6506	1.2069	1.8133
4	6.1	1.3863	1.8083	1.9218	2.5068
5	6.8	1.6094	1.9169	2.5903	3.0851
6	7.5	1.7918	2.0149	3.2104	3.6103
		$\Sigma X = 6.5792$	$\Sigma Y = 9.9319$	$\Sigma X^2 = 9.4099$	$\Sigma XY = 12.0201$

$$9.9319 = 6a + 6.5792b \rightarrow (A)$$

$$12.0201 = 6.5792a + 9.4099b \rightarrow (B)$$

Fit a best fitting curve $y = ax^b$

x (price)	20	16	10	11	14
y (demand)	22	41	120	89	56

Hence estimate demand (y) when price $x = 12$

$$y = ax^b$$

Applying $\log_e$ on both side

$$\log_e y = \log_e a (x)^b$$

$$= \log a + \log x^b$$

$$\log_e y = \log_e a + b \log_e x$$

$$Y = \log_e y \quad A = \log_e a \quad x = \log_e x$$

$$Y = A + bx$$

$$\Sigma y = an + b\Sigma x \rightarrow (1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow (2)$$

x	y	$x = \log x$	$y = \log y$	x^2	xy
20	22	2.9957	3.0910	8.9744	9.2597
16	41	2.7726	3.7136	7.6872	10.2963
10	120	2.3026	4.7875	5.3018	11.0237
11	89	2.3979	4.4886	5.7499	10.7632
14	56	2.6391	4.0254	6.9646	10.6234
		$\Sigma x =$ 13.1079	$\Sigma y =$ 20.1061	$\Sigma x^2 =$ 34.6783	$\Sigma xy =$ 51.9664

$$20.1061 = 5a + 13.1079b \rightarrow (A)$$

$$51.9664 = 13.1079a + 34.6783b \rightarrow (B)$$

Fit curve $y = ab^x$ for data

x	1	2	3	4	5	6	7
y	87	97	113	129	202	195	193

hence find y when $x = 8$

$$Y = ab^x$$

Applying $\log_e$ on both side

$$\log_e y = \log_e (a(b)^x)$$

$$= \log a + \log b^x$$

$$\log y = \log a + x \log b$$

$$Y = \log_e y \cdot A = \log a \quad B = \log_e b$$

$$\Sigma y = an + b\Sigma x \rightarrow (1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow (2)$$

x	y	$y = \ln y$	x^2	xy
1	87	4.4659	1	4.4659
2	97	4.5747	4	9.1494
3	113	4.7274	9	14.1822
4	129	4.8598	16	19.4392
5	202	5.3083	25	26.5415
6	195	5.2730	36	31.638
7	193	5.2627	49	36.8389
$\Sigma x =$ 28	~~$\Sigma y =$~~	$\Sigma y =$ 34.4715	$\Sigma x^2 =$ 140	$\Sigma xy =$ 142.2548

$34.4715 = 7a + 28b \rightarrow (A)$

$142.2548 = 28a + 140b \rightarrow (B)$

mul 4 into equ (A)

$137.886 = 28a + 112b$

$142.2548 = 28a + 140b$

$(-) \quad (-) \quad (-)$

$-4.3688 = -28b$

$\frac{4.3688}{28} = b$

$b = 0.1560$ sub in equ (A)

$34.4715 = 7a + 28b$

$34.4715 = 7a + 28(0.1560)$

$34.4715 = 7a + 4.368$

$34.4715 - 4.368 = 7a$

$30.1035 = 7a$

$\frac{30.1035}{7} = a$

$a = 4.3004$

$\log e^a = 4.3004$

$a = e^{4.3004}$

$a = 73.7293$

$\log e^b = 0.1560$

$b = e^{0.1560}$

$b = 1.1688$

$y = ab^x$

$= (73.7293)(1.1688)^x$

$x = 8$

$y = (73.7293)(1.1688)^8$

$y = 256.7803$

The population at a certain city at 10 years intervals is given by the following table

Years(x)	1941	1951	1961	1971	1981	1991	2001
Population(y)	3.9	5.3	7.3	9.6	12.9	17.1	23.2

Fit a curve of the form $y = ab^x$ to this data and estimate the population of the city in the year 2011.

$$y = ab^x$$

Applying $\log_e$ on both side

$$\log_e y = \log_e (ab^x)$$

$$\log y = \log a + x \log b$$

$$Y = A + Bx$$

$$Y = \log_e y \qquad A = \log_e a \qquad B = \log_e b$$

$$\Sigma y = an + b\Sigma x \rightarrow (1)$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \rightarrow (2)$$

$$n = 7$$

$$x_1 = x - 1941$$

x	y	$x = x-1941$	$Y = \log_e y$	x^2	xY
1941	3.9	0	1.3609	0	0
1951	5.3	10	1.6677	100	16.677
1961	7.3	20	1.9878	400	39.758
1971	9.6	30	2.2617	900	67.854
1981	12.9	40	2.5572	1600	102.288
1991	17.1	50	2.8391	2500	141.955
2001	23.2	60	3.1442	3600	188.652
		$\Sigma x =$ 210	$\Sigma Y =$ 15.8186	$\Sigma x^2 =$ 9100	$\Sigma xY =$ 557.179

$$15.8186 = 7a + 210b \rightarrow (A)$$

$$557.179 = 210a + 9100b \rightarrow (B)$$

mul 30 in equ (A)

$$474.558 = 210a + 6300b$$

$$557.179 = 210a + 9100b$$

$$-82.621 = -2800b$$

$$\frac{82.621}{2800} = b$$

$b = 0.0295075$ sub in equ (A)

$$15.8186 = 7a + 210b$$

$$15.8186 = 7a + 210(0.0295075)$$

$$15.8186 = 7a + 6.196575$$

$$15.8186 - 6.196575 = 7a$$

$$9.622025 = 7a$$

$$\frac{9.622025}{7} = a$$

$$a = 1.374575$$

$\log e^{a} = 1.374575$

$a = e^{1.374575}$

$a = 3.9534$

$\log e^{b} = 0.0295075$

$b = e^{0.0295075}$

$b = 1.0299$

$$y = ab^{x}$$

$$y = 3.9534\,(1.0299)^{x-1941}$$

$x = 2011$

$$y = 3.9534\,(1.0299)^{2011-1941}$$

$$= 3.9534 \times 1.0299^{70}$$

$$y = 31.09029394$$