MARUDHAR KESARI JAIN COLLEGE FOR WOMEN,VANIYAMBADI
PG & RESEARCH DEPARTMENT OF MATHEMATICS

SUBJECT NAME: NUMERICAL METHODS I

CLASS : 1 B.Sc CS

CODE: 23UECS12A

SYLLABUS:

Unit-V Central differences formulae Operators, and relation with the other operators. Gauss forward and backward formulae, Stirling's formula and Bessel's formula

Unit - 2

Solution of Simultaneous linear equation

* Gauss Elimination method.

Taking the system of n linear equations $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}a_n = b_n \rightarrow (1)$$

This method is based on the system of equation reduced to triangular form by successive elimination of variable

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1) Solve the equation by using gauss elimination method

$$x_1 + 2x_2 + x_3 = 8$$

$$2x_1 + 3x_2 + 4x_3 = 20$$

$$4x_1 + 3x_2 + 2x_3 = 16$$

The equation can be written as $AX = B$

$$AX = B \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 4 \\ 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 20 \\ 16 \end{bmatrix}$$

The argumented matrix

$$[A, B] \sim \begin{bmatrix} 1 & 2 & 1 & 8 \\ 2 & 3 & 4 & 20 \\ 4 & 3 & 2 & 16 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & -1 & 2 & 4 \\ 0 & -5 & -2 & -16 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 8 \\ 0 & -1 & 2 & 4 \\ 0 & 0 & -12 & -36 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - 5R_2 \end{matrix}$$

$x_1 + 2x_2 + x_3 = 8 \longrightarrow (1)$

$-x_2 + 2x_3 = 4 \longrightarrow (2)$

$-12x_3 = -36 \longrightarrow (3)$

$$x_3 = \frac{-36}{-12} = 3$$

$$\boxed{x_3 = 3}$$

(2) $\Rightarrow -x_2 + 2(3) = 4$

$-x_2 = -2$

$$\boxed{x_2 = 2}$$

Sub x_2 & x_3 in (1)

$x_1 + 4 + 3 = 8$

$$\boxed{x_1 = 1}$$

The Solutions are $x_1 = 1$, $x_2 = 2$ and $x_3 = 3$

2) $2x + 3y - z = 5, \; 4x + 4y - 3z = 3, \; 2z - 3y + 2z = 2$

The given equation is from $AX = B$

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -3 \\ 2 & -3 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 3 \\ 2 \end{bmatrix}$$

the argument matrix

$$[A/B] \sim \begin{bmatrix} 2 & 3 & -1 & 5 \\ 4 & 4 & -3 & 3 \\ 2 & -3 & 2 & 2 \end{bmatrix} C_1 \to C_1 \div 2$$

$$\sim \begin{bmatrix} 1 & 3 & -1 & 5 \\ 2 & 4 & -3 & 3 \\ 1 & -3 & 2 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 2 & -1 & -7 \\ 0 & -6 & 3 & -3 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & -2 & -1 & -7 \\ 0 & 0 & 6 & 18 \end{bmatrix} R_3 \to R_3 - 3R_2$$

$$x + 3y - z = 5$$

$$-2y - z = -7$$

$$6z = 18$$

$$z = 18/6$$

$$\boxed{z = 3}$$

Sub z values in eq 2

$$-2y = -7 + 3$$

$$\boxed{y = 2}$$

Sub y, z in eq(1)

$$x + 6 - 3 = 5$$

$$x + 3 = 5$$

$$\boxed{x = 2}$$

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3) $x + 2y + z = 3$, $2x + 3y + 3z = 10$,

$3x - 2y + 2z = 13$

The given equation is from $AX = B$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & -2 & 2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 10 \\ 13 \end{bmatrix}$$

The argument matrix

$$[A/B] \sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -2 & 2 & 13 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\begin{matrix} 2 & 3 & 3 & 10 \\ -2 & -4 & -2 & -6 \\ \hline 0 & -1 & 1 & 4 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -8 & -1 & 4 \end{bmatrix} R_3 \to R_3 - 8R_2$$

$$\begin{matrix} 3 & -2 & 2 & 13 \\ -3 & -6 & -3 & -9 \\ \hline 0 & -8 & -1 & 4 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{bmatrix}$$

$$\begin{matrix} 0 & -8 & -1 & 4 \\ 0 & 8 & -8 & -32 \\ \hline 0 & 0 & -8 & -24 \end{matrix}$$

$$x + 2y + z = 3 \quad — (1)$$

$$-y + z = 4 \quad — (2)$$

$$-8z = -24$$

$$\boxed{z = 3} \quad — (3)$$

Sub $z=3$ in ②

$-y+z=4$

$-y+3=4$

$-y=1$

$\boxed{y=-1}$

Sub $z=3,\ y=-1$ in ①

$2+2(-1)+3=3$

$2+1=3$

$3=3$

$x+2y+z=3$

$x+2(-1)+3=3$

$x=3-1$

$\boxed{x=2}$

$\therefore$ Result $\Rightarrow$ The values are $x=2$, $y=-1$ and $z=3$

Gauss Jordon Method

* This method is modified form of gauss elimination method
* The coefficient of matrix of System $ax=b$ is brought to diagonal matrix or unit matrix
* By matrix A not only by upper triangular matrix by also by lower trianglar matrix

$$\left[\begin{array}{cc|c} a_{11} & 0000 & d_1 \\ 0 & b_{22}\ 000 & d_2 \\ \vdots & & \\ 0 & 000a_{mn} & d_n \end{array}\right]$$

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(1) Solve by using Gauss Jordon method

$x+2y+z=3,\ 2x+3y+2z=10,\ 3x-y+2z=13$

The equation can be written $AX=B$

$$A=\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & -1 & 2 \end{bmatrix} \quad X=\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B=\begin{bmatrix} 3 \\ 10 \\ 13 \end{bmatrix}$$

$-26+(-12)+48$

Argumented matrix

$$[A,B]=\begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 3 & 2 & 10 \\ 3 & -1 & 2 & 13 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & 4 \\ 0 & -7 & -1 & 4 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & -1 & -24 \end{bmatrix} R_3 \to R_3 - 7R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & -21 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & -1 & -24 \end{bmatrix} R_1 \to R_1 + R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & -13 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & -1 & -24 \end{bmatrix} R_1 \to R_1 + 2R_2$$

$x + 2y + z = 3$

$-13 - 8 + 24 = 3$

$-21 + 24 = 3$

$3 = 3$

$x = -13$

$-y = 4$ (ie) $y = -4$

$-z = -24$ (ie) $z = 24$

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1) Solve the equation by using Gauss Jordan method $x + 2y + z = 3$, $2x + 3y + 2z = 10$, $3x - y + 2z = 13$

Argumented matrix

$$[A, B] \sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 2 & 13 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & -1 & 4 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -8 & -24 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3 - 7R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 3 & 11 \\ 0 & -8 & 0 & 8 \\ 0 & 0 & -8 & -24 \end{bmatrix} \begin{matrix} R_1 \to R_1 + 2R_2 \\ R_2 \to R_3 + 8R_2 \\ \\ \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 3 & 11 \\ 0 & -8 & 0 & 8 \\ 0 & 0 & -1 & -3 \end{bmatrix} \begin{matrix} \\ \\ R_3 \to R_3/8 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & -8 & 0 & 8 \\ 0 & 0 & -1 & -3 \end{bmatrix} \begin{matrix} R_1 \to R_1 + 3R_3 \\ \\ \\ \end{matrix}$$

$$\boxed{x = 2}$$

$$-8y = 8$$

$$\boxed{y = -1}$$

$$-z = -3$$

$$\boxed{z = 3}$$

$\therefore$ The required values are $x = 2$, $y = -1$, $z = 3$

2) Solve by Gauss Jordan method

$$x + y + z + w = 2$$

$$2x - y + 2z - w = -5$$

$$3x + 2y + 3z + 4w = 7$$

$$x - 2y - 3z + 2w = 5$$

The equation can be written as $AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -1 & 2 & -1 \\ 3 & 2 & 3 & 4 \\ 1 & -2 & -3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ -5 \\ 7 \\ 5 \end{bmatrix}$$

Argumented matrix

$$[A, B] \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 2 & -1 & 2 & -1 & -5 \\ 3 & 2 & 3 & 4 & 7 \\ 1 & -2 & -3 & 2 & 5 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & -3 & -4 & 1 & 3 \end{bmatrix} \quad R_2 \to \frac{R_2}{-3}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 2 & 4 \\ 0 & 0 & -4 & -2 & 0 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 + R_2 \\ R_4 \to R_4 - 3R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 & 0 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3/2 \\ R_4 \to R_4/-2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 & 0 \end{bmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0 & 2 \\ 0 & 0 & 2 & 1 & 0 \end{bmatrix} R_3 \to R_3 - R_4$$

$$\sim \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & -2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \begin{matrix} R_1 \to 2R_1 + R_3 \\ R_4 \to R_4 + R_3 \end{matrix}$$

$x = 0$ $\quad$ $-2z = 2$ $\quad$ $\boxed{w = 2}$

$\boxed{y = 1}$ $\quad$ $\boxed{z = -1}$

$\therefore$ The required values are $x = 0, y = 1,$

$z = -1$ and $w = 2$

Gauss Seidel method of Iteration

* This method is applicable when each equation of system contains one coefficient much larger than other coefficent of equation

* This condition will be statisfied if the largest coefficient are along the leading diagonal of the coefficient matrix

* When this condition is satisfied the system will be solvable by the iteration method.

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

will be solvable if

$$|a_{11}| > |a_{12}| + |a_{13}|$$

$$|a_{22}| > |a_{21}| + |a_{23}|$$

$$|a_{33}| > |a_{31}| + |a_{32}|$$

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1. Solve the following System by Gauss Seidel method $10x - 5y - 2z = 3$, $4x - 10y + 3z = -3$, $x + 6y + 10z = -3$

$$10x - 5y - 2z = 3 \rightarrow (1)$$

$$4x - 10y + 3z = -3 \rightarrow (2)$$

$$x + 6y + 10z = -3 \rightarrow (3)$$

$$|a_{11}| > |a_{12}| + |a_{13}|$$

$$10 > 7$$

$$|a_{22}| > |a_{21}| + |a_{23}|$$

$$10 > 7$$

$$|a_{33}| > |a_{31}| + |a_{32}|$$

$$10 > 7$$

$\therefore$ The conditions are Statisfied.

$$(1) \Rightarrow 10x = 3 + 5y + 2z$$

$$x = \frac{1}{10}(3 + 5y + 2z) \Rightarrow (4)$$

$$(2) \Rightarrow -10y = -3-4x-3z$$

$$y = \frac{1}{10}(3+4x+3z) \rightarrow (5)$$

$$(3) \Rightarrow 10z = -3-x-6y$$

$$z = \frac{1}{10}(-3-x-6y) \rightarrow (6)$$

Initial value, $y=0, z=0$

$$(4) \Rightarrow x^{(1)} = \frac{1}{10}(3+0+0)$$

$$x^{(1)} = \frac{3}{10} = 0.3$$

$$y^{(1)} = \frac{1}{10}(3+4(0.3)+0)$$

$$= \frac{1}{10}(3+1.2) = \frac{1}{10}(4.2)$$

$$y^{(1)} = 0.42$$

$$z^{(1)} = \frac{1}{10}(-3-0.3-6(0.42)]$$

$$= \frac{1}{10}(-5.82)$$

$$z^{(1)} = -0.582$$

2nd iteration

$$x^{(2)} = \frac{1}{10}(3+5(0.42)+2(-0.582)]$$

$$= \frac{1}{10}(3.936)$$

$$x^{(2)} = 0.3936$$

$$y^{(2)} = \frac{1}{10}[3+4(0.3936)+3(-0.582)$$

$$= \frac{1}{10}(2.8284) = 0.28284$$

$z^{(2)} = \frac{1}{10}(-3 - 0.3936 - 6(0.28284))$

$= (-5.09064)\frac{1}{10}$

$0.3414849 = x^{(1)}$

$0.28504212 = y^{(1)}$

$-0.5051731 = z^{(1)}$

$z^{(2)} = -0.509064$

3rd iteration

$x^{(3)} = \frac{1}{10}(3 + 5(0.28284) + 2(-0.509064))$

$= \frac{1}{10}[3.396072]$

$= 0.3396072$

$y^{(3)} = \frac{1}{10}[3 + 4(0.3396072) + 3(-0.509064)]$

$= (2.8312368)\frac{1}{10}$

$y^{(3)} = 0.28312368$

$z^{(3)} = \frac{1}{10}(-3 - 0.3396072 - 6(0.28312368))$

$z^{(3)} = (-5.03834928)\frac{1}{10}$

$z^{(3)} = -0.503834928$

4th iteration

$x^{(4)} = \frac{1}{10}[3 + 5(0.28312368) + 2(-0.503834928)]$

$x^{(4)} = 0.340794854$

$$\cancel{y^{(4)} = \frac{1}{10}(3+4(0.28312368) +}$$

$$y^{(4)} = \frac{1}{10}\left[3+4(0.3407948544) + 3(-0.503834928)\right]$$

$$= \frac{1}{10}\left[2.851674634\right]$$

$$y^{(4)} = 0.2851674634$$

$$z^{(4)} = \frac{1}{10}\left[-3 - 0.3407948544 - 6(0.2851674634)\right]$$

$$= (-5.051799635)\frac{1}{10}$$

$$z^{(4)} = -0.5051799635$$

5th iteration

$$x^{(5)} = \frac{1}{10}\left(3+5(0.2851674634) + 2(-0.5051799635)\right)$$

$$= 0.3.41547739$$

$$y^{(5)} = \frac{1}{10}\left(3+4(0.341547739)+3(-0.5051799635)\right)$$

$$y^{(5)} = 0.2850651066$$

$$z^{(5)} = \frac{1}{10}\left(-3-0.341547739-6(0.2850651066)\right)$$

$$z^{(5)} = -0.5051938379$$

6^{th} iteration

$$x^{(6)} = \frac{1}{10}(3+5(0.2850651066)+2(-0.5051938379)$$

$$= 3.414937857 = 0.3414937857$$

$$y^{(6)} = \frac{1}{10}(3+4(0.3414937857)+3(-0.5051938379)$$

$$y^{(6)} = 0.2850393629$$

$$z^{(6)} = \frac{1}{10}(-3-0.3414937857-6(0.2850393629)$$

$$z^{(6)} = -0.5051729963$$

7^{th} Iteration

$$x^{(7)} = \frac{1}{10}(3+5(0.2850393629)+2(-0.5051729963))$$

$$x^{(7)} = 0.3414850822$$

$$\cancel{x}y^{(7)} = \frac{1}{10}(+3+4(0.3414850822)+3(-0.5051729963))$$

$$y^{(7)} = 0.285042134$$

$$z^{(7)} = \frac{1}{10}(-3-0.3414850822-6(0.285042134))$$

$$z^{(7)} = -0.5051737886$$

8^{th} Iteration

$$x^{(8)} = \frac{1}{10}\left(3+5(0.285042134)+ 2(-0.5051737886)\right)$$

$$x^{(8)} = 0.3414863093$$

$$y^{(8)} = \frac{1}{10}\left(3+4(0.3414863093)+ 3(-0.5051737886)\right)$$

$$y^{(8)} = 0.2850423871$$

$$z^{(8)} = \frac{1}{10}\left(-3-0.3414863093- 6(0.2850423871)\right)$$

$$z^{(8)} = -0.5051740632$$

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2 Solve the following System by Gauss Seidel method [Correct three decimals] $8x-3y+2z=20$, $4x+11y-z=33$, $6x+3y+12z=35$

$$8x-3y+2z=20 \longrightarrow (1)$$

$$4x+11y-z=33 \longrightarrow (2)$$

$$6x+3y+12z=35$$

$$|a_{11}| > |a_{12}|+|a_{13}|$$

$$8 > 5$$

$$|a_{22}| > |a_{21}|+|a_{23}|$$

$$14 > 5$$

$$|a_{33}| > |a_{31}|+|a_{32}|$$

$$12 > 8$$

$8x - 3y + 2z = 20$

$$x = \frac{1}{8}[20 + 3y - 2z] \quad \text{--- (4)}$$

$4x + 11y - z = 33$

$$y = \frac{1}{11}[33 - 4x + z] \quad \text{--- (5)}$$

$6x + 3y + 12z = 35$

$$z = \frac{1}{12}[35 - 6x - 3y] \quad \text{--- (6)}$$

<u>Initial value . y=0, z=0</u>

$$(4) \Rightarrow x^{(1)} = \frac{1}{8}[20] = 2.5$$

$$y^{(1)} = \frac{1}{11}[33 - 4(2.5) + 0]$$

$$= \frac{23}{11} = 2.090909$$

$$z^{(1)} = \frac{1}{12}[35 - 6(2.5) - 3(2.090909)]$$

$$= \frac{13.727273}{12} = 1.143939417$$

<u>2nd Iteration</u>

$$(x)^{(2)} = \frac{1}{8}[20 + 3(2.090909) - 2(1.143939417)]$$

$$= \frac{23.98484817}{8}$$

$$x^{(2)} = 2.998106021$$

$$y^{(2)} = \frac{1}{11}[33 - 4(2.998106021) + 1.143939417]$$

$$y^{(2)} = \frac{22.15151533}{11} = 2.013774121$$

$$z^{(2)} = \frac{1}{12}(35 - 6(2.998106021) - 3(2.013774121))$$

$$= \frac{10.97004151}{12} = 0.914170125$$

3rd Iteration

$$x^{(3)} = \frac{1}{8}[20 + 3(2.013774121) - 2(0.914170125))$$

$$= \frac{24.21298211}{8} = 3.026622764$$

$$y^{(3)} = \frac{1}{11}[33 - 4(3.026622764) + 0.914170125]$$

$$y^{(3)} = \frac{21.80767907}{11} = 1.982516279$$

$$z^{(3)} = \frac{1}{12}[35 - 6(3.026622764) - 3(1.982516279))$$

$$= \frac{10.89271458}{12}$$

$$z^{(3)} = 0.907726215$$

4^{th} Iteration

$$x^{(4)} = \frac{1}{8}\left[20+3(1.982516279)-2(0.907726215)\right.$$

$$x^{(4)} = 3.016512051$$

$$y^{(4)} = \frac{1}{11}\left(33-4(3.016512051)+0.907726215\right]$$

$$y^{(4)} = 1.985607092$$

$$z^{(4)} = \frac{1}{12}\left(35-6(3.016512051)-3(1.985607092)\right)$$

$$z^{(4)} = 0.912008868$$

5^{th} Iteration

$$x^{(5)} = \frac{1}{8}\left[20+3(1.985607092)-2(0.912008868)\right]$$

$$x^{(5)} = 3.016600443$$

$$y^{(5)} = \frac{1}{11}\left(33-4(3.016600443)+0.912008868\right)$$

$$y^{(5)} = 1.985964281$$

$$z^{(5)} = \frac{1}{12}\left[35-6(3.016600443)-3(1.985964281)\right.$$

$$z^{(5)} = 0.911875374$$

6^{th} Iteration

$$x^{(6)} = \frac{1}{8}\left[20+3(1.985964281)-2(0.911875374)\right]$$

$$= 3.016767762$$

$$y^{(6)} = \frac{1}{11}\left[33-4(3.016767762)+0.911875374\right]$$

$$y^{(6)} = 1.985891302$$

$$z^{(6)} = \frac{1}{12}\left[35-6(3.016767762)-3(1.985891302)\right]$$

$$z^{(6)} = 0.91180996$$

7^{th} Iteration

$$x^{(7)} = \frac{1}{8}\left[20+3(1.985891302)-2(0.91180996)\right]$$

$$x^{(7)} = 3.016756748$$

$$y^{(7)} = \frac{1}{11}\left[33-4(3.016756748)+0.91180996\right]$$

$$y^{(7)} = 1.985889361$$

$$z^{(7)} = \frac{1}{12}\left[35-6(3.016756748)-3(1.985889361)\right]$$

$$z^{(7)} = 0.911815952$$

$\therefore$ The values are

$$x = 3.01676$$
$$y = 1.98589$$
$$z = 0.91181$$

3) Solve the following System by Gauss Seidal method correct to three decimal places.

$$x + y + 54z = 110$$

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

$$|a_{11}| > |a_{12}| + |a_{13}|$$

$$27 > 7$$

$$|a_{22}| > |a_{21}| + |a_{23}|$$

$$15 > 8$$

$$|a_{33}| > |a_{31}| + |a_{32}|$$

$$54 > 2$$

$\therefore$ The condition statisfied

$$27x + 6y - z = 85 \rightarrow (1)$$

$$6x + 15y + 2z = 72 \rightarrow (2)$$

$$x + y + 54z = 110 \rightarrow (3)$$

$$x = \frac{1}{27}[-6y + z + 85] - (4)$$

$$y = \frac{1}{15}[72 - 6x - 2z] - (5)$$

$$z = \frac{1}{54}[110 - x - y] - (6)$$

<u>Initial Value $y=0, z=0$</u>

$$x^{(1)} = \frac{1}{27}[85]$$

$$x^{(1)} = 3.148148$$

$$y^{(1)} = \frac{1}{15}[72-6(3.148148)-2(0)]$$

$$y^{(1)} = 3.5407408$$

$$z^{(1)} = \frac{1}{54}[110-3.148148-3.5407408]$$

$$z^{(1)} = 1.9131687$$

<u>2^{nd} Iteration</u>

$$x^{(2)} = \frac{1}{27}[85-6(3.5407408)+\cancel{2}(1.9131687)]$$

$$x^{(2)} = 2.4321749$$

$$y^{(2)} = \frac{1}{15}[72-6(2.4321749)-2(1.9131687)]$$

$$= 3.5720408$$

$$z^{(2)} = \frac{1}{54}[110-2.4321749-3.5720408]$$

$$z^{(2)} = 1.9258478$$

3rd Iteration

$$x^{(3)} = \frac{1}{27}[85 - 6(3.5720408) + 1.9258478]$$

$$x^{(3)} = 2.425689$$

$$y^{(3)} = \frac{1}{15}[72 - 6(2.425689) - 2(1.9258478)]$$

$$y^{(3)} = 3.5729446$$

$$z^{(3)} = \frac{1}{54}[110 - 2.425689 - 3.5729446]$$

$$z^{(3)} = 1.9259512$$

4th Iteration

$$x^{(4)} = \frac{1}{27}[85 - 6(3.5729446) + 1.9259512]$$

$$x^{(4)} = 2.4254919$$

$$y^{(4)} = \frac{1}{15}[72 - 6(2.4254919) - 2(1.9259512)]$$

$$y^{(4)} = 3.5730097$$

$$z^{(4)} = \frac{1}{54}[110 - 2.4254919 - 3.5730097]$$

$$z^{(4)} = 1.92595374$$

5^{th} Iteration

$x^{(5)} = \frac{1}{27}\left[85 - 6(3.5730097) + 1.9259536\right]$

$x^{(5)} = 2.4254776$

$y^{(5)} = \frac{1}{15}\left[72 - 6(2.4254776) - 2(1.9259536)\right]$

$y^{(5)} = 3.57301514$

$z^{(5)} = \frac{1}{54}\left[110 - 2.4254776 - 3.57301514\right]$

$z^{(5)} = 1.92595538$

6^{th} Iteration.

$x^{(6)} = \frac{1}{27}\left[85 + z - 6y\right]$

$= \frac{1}{27}\left[85 - 6(3.57301514) + 1.9259538\right]$

$x^{(6)} = 2.425476$

$y^{(6)} = \frac{1}{15}\left[72 - 6(2.425476) - 2(1.9259538)\right]$

$y^{(6)} = 3.5730157$

$z^{(6)} = \frac{1}{54}\left[110 - 2.425476 - 3.5730157\right]$

$z^{(6)} = 1.9259538$

$\therefore$ The values are $x = 2.42547$

$y = 3.573015$

$z = 1.92595$

4) Solve the following system by Gauss Seidal method correct the decimal places.

$x = 0.9936$
$y = 1.5069$
$z = 1.8486$

$$28x + 4y - z = 32$$
$$x + 3y + 10z = 24$$
$$2x + 17y + 4z = 35$$

$|a_{11}| > |a_{12}| + |a_{13}|$

$28 > 5$

$|a_{22}| > |a_{21}| + |a_{23}|$

$17 > 6$

$|a_{33}| > |a_{31}| + |a_{32}|$

$10 > 4$

$\therefore$ The condition is satisfied

$$28x + 4y - z = 32 \quad -① $$
$$x + 3y + 10z = 24 \quad -③$$
$$2x + 17y + 4z = 35 \quad -②$$

$$x = \frac{1}{28}[32 - 4y + z] \quad -④$$

$$y = \frac{1}{17}[35 - 2x - 4z] \quad -⑤$$

$$z = \frac{1}{10}[24 - x - 3y] \quad -⑥$$

Initial value $y = 0, z = 0$

$$x^{(1)} = \frac{1}{28}[32] = 1.14285714$$

$$y^{(1)} = \frac{1}{17}[35 - 2(1.14285714) - 4(0)]$$

$$y^{(1)} = 1.9243697$$

$$z^{(1)} = \frac{1}{10}[24 - 1.14285714 - 3(1.9243697)]$$

$$z^{(1)} = 1.7084033$$

2nd Iteration

$$x^{(2)} = \frac{1}{28}[32 - 4(1.9243697) + 1.7084033]$$

$$x^{(2)} = 0.9289615$$

$$y^{(2)} = \frac{1}{17}[35 - 2(0.9289615) - 4(1.7084033)]$$

$$y^{(2)} = 1.54755669$$

$$z^{(2)} = \frac{1}{10}[24 - 0.9289615 - 3(1.54755669)]$$

$$z^{(2)} = 1.84283684$$

3rd Iteration

$$x^{(3)} = \frac{1}{28}[32 - 4(1.54755669) + 1.84283684]$$

$$x^{(3)} = 0.9875932$$

$$y^{(3)} = \frac{1}{17}[35 - 2(0.9875932) - 4(1.84283684)]$$

$$y^{(3)} = 1.5090274$$

Address

$$z^{(3)} = \frac{1}{10}[24 - x - 3y]$$

$$= \frac{1}{10}[24 - 0.9875932 - 3(1.5090274)]$$

$$z^{(3)} = 1.84853246$$

4th Iteration

$$x^{(4)} = \frac{1}{28}[32 - 4(1.5090274) + 1.84853246]$$

$$x^{(4)} = 0.9933008$$

$$y^{(4)} = \frac{1}{17}[35 - 2(0.9933008) - 4(1.\cancel{8}4853246)]$$

$$= 1.50701579$$

$$z^{(4)} = \frac{1}{10}[24 - 0.9933008 - 3(1.50701579)]$$

$$z^{(4)} = 1.84856518$$

5th Iteration

$$x^{(5)} = \frac{1}{28}[32 - 4(1.50701579) + 1.84856518]$$

$$x^{(5)} = 0.99358935$$

$$y^{(5)} = \frac{1}{17}[35 - 2(0.99358935) - 4(1.84856518)]$$

$$y^{(5)} = 1.50697415$$

$z^{(5)} = \frac{1}{10}[24 - 0.99358935 - 3(1.50697415)]$

$z^{(5)} = 1.8485488$

6th Iteration

$x^{(6)} = \frac{1}{28}[32 - 4(1.50697415) + 1.8485488]$

$x^{(6)} = 0.99359472$

$y^{(6)} = \frac{1}{17}[35 - 2(0.99359472) - 4(1.8485488)]$

$y^{(6)} = 1.5069773$

$z^{(6)} = \frac{1}{10}[24 - 0.99359472 - 3(1.5069773)]$

$z^{(6)} = 1.848547338$

7th Iteration

$x^{(7)} = \frac{1}{28}[32 - 4(1.5069773) + 1.848547338]$

$x^{(7)} = 0.99359421$

$y^{(7)} = \frac{1}{17}[35 - 2(0.99359421) - 4(1.848547338)]$

$y^{(7)} = 1.50697777$

$z^{(7)} = \frac{1}{10}[24 - 0.99359421 - 3(1.50697777)]$

$z^{(7)} = 1.848547248$

The values are

$x = 0.993594$

$y = 1.506977$

$z = 1.848547$

Crout's method (Ducot method)

consider the System $AX = B$

where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

Suppose we decomposed $A = LU$

where $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$, $U = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$

Since $AX = B$

$= LUX = B$

$LY = B$ where $UX = Y$

$\Rightarrow LU = A$ Reduces to

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

(ie) $$\begin{bmatrix} l_{11} & l_{11}U_{12} & l_{11}U_{13} \\ l_{21} & l_{21}U_{12} + l_{22} & l_{21}U_{13} - l_{22}U_{23} \\ l_{31} & l_{31}U_{22} + l_{32} & l_{31}U_{13} + l_{32}U_{23} + l_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Equation co-efficient and simplifying, we get

$l_{11} = a_{11}, \; l_{12} = a_{21}, \; l_{13} = a_{31}$

$\Rightarrow l_{11} \, a_{12} = a_{12}$	$l_{11} u_{13} = a_{13}$
$a_{11} \, u_{12} = a_{12}$	$a_{11} u_{13} = a_{13}$
$u_{12} = \dfrac{a_{12}}{a_{11}}$	$U_{13} = \dfrac{a_{13}}{a_{11}}$

$$l_{21} u_{12} + l_{22} = a_{22}$$

$$l_{22} = a_{22} - l_{21} u_{12}$$

$$l_{21} u_{13} + l_{22} u_{23} = a_{23}$$

$$l_{22} u_{23} = a_{23} - l_{21} u_{13}$$

$$u_{23} = \frac{a_{23} - l_{21} u_{13}}{l_{22}}$$

$$l_{31} u_{22} + l_{32} = a_{32}$$

$$l_{32} = a_{32} - l_{31} u_{22}$$

$$l_{31} u_{13} - l_{32} u_{23} + l_{33} = a_{33}$$

$$l_{33} = a_{33} - l_{31} u_{13} - l_{32} u_{23}$$

Now L and U are known

$$LY = B$$

$$\begin{bmatrix} l_{11} y_1 \\ l_{21} y_1 + l_{22} y_2 \\ l_{31} y_1 + l_{32} y_2 + l_{33} y_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Equating co-efficient

$l_{11} y_1 = b_1$

$$y_1 = \frac{b_1}{l_{11}} = \frac{b_1}{a_{11}}$$

$l_{21} y_1 + l_{22} y_2 = b_2$

$$l_{22} y_2 = b_2 - l_{21} y_1$$

$$y_2 = \frac{b_2 - l_{21} y_1}{l_{22}}$$

$l_{31} y_1 + l_{32} y_2 + l_{33} y_2 = b_3$

$$l_{33} y_3 = b_3 - l_{31} y_1 - l_{32} y_2$$

$$y_3 = \frac{b_3 - l_{31} y_1 - l_{32} y_2}{l_{33}}$$

$\Rightarrow$ y is formed

Derived matrix = $\begin{bmatrix} l_{11} & u_{12} & u_{13} & y_1 \\ l_{21} & l_{22} & u_{23} & y_2 \\ l_{31} & l_{32} & l_{33} & y_3 \end{bmatrix}$

5m

If we know the derived matrix, we can write L, U and Y. The derived matrix is got as explained below, using the argumented matrix (A, B)

STEP I : The first column of D.M (derived matrix) is the same as the first column of A

STEP 2 : The remaining elements of first row of D.M. Each elements of the row of D.M (except the first element l_{11}) is got by dividing the corresponding element in (A, B) by the leading diagonal element of that row.

STEP 3 : Remaining elements of second column of D.M. Since,

$$l_{22} = a_{22} - l_{21} u_{12} ; l_{32} = a_{32} - l_{31} u_{12}$$

Each element of second column except top element } = Corresponding element in (A, B) minus the product of the first element in that row and in that column.

STEP 4 : Remaining elements of second row

Each element = corresponding element in (A, B) minus sum of the inner product of the previously calculated elements in the same row and same column divided by diagonal elements in that row.

STEP 5 : Remaining elements of third column

$$l_{33} = a_{33} - l_{31} u_{13} - l_{32} u_{23}$$

The element = corresponding elements in (A, B) minus sum of the inner product of the previously calculated in the same rows and columns.

STEP 6 : Remaining elements of third row.

$$y_3 = \frac{b_3 - (l_{31} y_1 + l_{32} y_2)}{l_{33}}$$

The element - (corresponding element of A, B) - (sum of the inner products of previously calculated elements in the same rows and columns) divided by a diagonal element of that row.

1. Solve by using crout's method

$x + y + z = 3$

$2x - y + 3z = 16$

$3x + y - z = -3$

$$(A,B) = \begin{bmatrix} \overset{a_{11}}{1} & \overset{a_{12}}{1} & \overset{a_{13}}{1} & \overset{b_1}{3} \\ \overset{a_{21}}{2} & \overset{a_{22}}{-1} & \overset{a_{23}}{3} & \overset{b_2}{16} \\ \overset{a_{31}}{3} & \overset{a_{32}}{1} & \overset{a_{33}}{-1} & \overset{b_3}{-3} \end{bmatrix}$$

$$D.M = \begin{bmatrix} l_{11} & u_{12} & u_{13} & y_1 \\ l_{21} & l_{22} & u_{23} & y_2 \\ l_{31} & l_{32} & l_{33} & y_3 \end{bmatrix}$$

Step 1 : Element of 1st column of D.M

$$D.M = \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ 2 & \cdot & \cdot & \cdot \\ 3 & \cdot & \cdot & \cdot \end{bmatrix}$$

Step 2 : Element of 1st row of D.M

$u_{12} = \frac{1}{1} = 1$; $u_{13} = \frac{1}{1} = 1$; $y_1 = \frac{3}{1} = 3$

$$DM = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & . & . & . \\ 3 & . & . & . \end{bmatrix}$$

Step 3 : Elements of second colum of D.M

$l_{22} = a_{22} - l_{21} u_{12} = -1 - 2(1) = -3$

$l_{32} = a_{32} - l_{31} u_{12} = 1 - 3(1) = -2$

$$D.M = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -3 & . & . \\ 3 & -2 & . & . \end{bmatrix}$$

Step 4 : Element of 2nd row of D.M

$$u_{23} = \frac{a_{23} - l_{21} u_{13}}{l_{22}} = \frac{3 - 2(1)}{-3}$$

$$u_{23} = \frac{1}{-3} = -1/3$$

$$y_2 = \frac{b_2 - l_{21} y_1}{l_{22}} = \frac{16 - (2)(3)}{-3} = \frac{10}{-3} = \frac{-10}{3}$$

$$D.M = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -3 & -1/3 & -10/3 \\ 3 & -2 & . & . \end{bmatrix}$$

Step 5 : Element of 3rd column in D.M

$l_{33} = a_{33} - l_{31} u_{13} - l_{32} u_{23}$

$= -1 - 3(1) - (-2)(-1/3) = -1 - 3 + (-2/3)$

$= -4 - 2/3 = \frac{-12 - 2}{3} = \frac{-14}{3}$

$$D.M = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -3 & -1/3 & 3 \\ 3 & -2 & -14/3 & . \end{bmatrix}$$

Step 6 : Element of 3^{rd} row in D.M

$$y_3 = \frac{b_3 - (l_{31} y_1 + l_{32} y_2)}{l_{33}}$$

$$= \frac{-3 - [3 \times 3 + (-2)(-10/3)]}{14/3}$$

$$y_3 = \frac{-3 - (9 + 20/3)}{-14/3} = \left(-3 - 9 - \frac{20}{3}\right)\left(\frac{-3}{14}\right)$$

$$\boxed{y_3 = 4}$$

$$D.M = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -3 & -1/3 & -10/3 \\ 3 & -2 & -14/3 & 4 \end{bmatrix}$$

The Solution $Ux = y$

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -10/3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1/3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -10/3 \\ 4 \end{bmatrix}$$

$x + y + z = 3 \longrightarrow (1)$

$y - \frac{1}{3}z = -\frac{10}{3} \longrightarrow (2)$

$\boxed{z = 4}$

Sub z in (2)

$y - \frac{1}{3}(4) = \frac{-10}{3}$

$y - \frac{4}{3} = \frac{-10}{3}$

$y = \frac{-10}{3} + \frac{4}{3}$

$\boxed{y = -2}$

(1) $\Rightarrow x - 2 + 4 = 3$

$x = 3 - 2$

$\boxed{x = 1}$

$\therefore$ The values are $x = 1, y = -2, z = 4$

---

2 Solve the System of equation by crout's method

$2x + 3y + z = -1$

$5x + y + z = 9$

$3x + 2y + 4z = 11$

$$[A, B] = \begin{bmatrix} 2 & 3 & 1 & -1 \\ 5 & 1 & 1 & 9 \\ 3 & 2 & 4 & 11 \end{bmatrix}$$

$$D.M = \begin{bmatrix} l_{11} & u_{12} & u_{13} & y_1 \\ l_{21} & l_{22} & u_{23} & y_2 \\ l_{31} & l_{32} & l_{33} & y_3 \end{bmatrix}$$

<u>STEP 1</u> : Elements of 1st column of D.M

$$D.M = \begin{bmatrix} 2 & . & . & . \\ 5 & . & . & . \\ 3 & . & . & . \end{bmatrix}$$

<u>STEP 2</u> : Elements of 1st row of D.M

$u_{12} = \frac{3}{2}$, $u_{13} = \frac{1}{2}$, $y_1 = -\frac{1}{2}$

$$D.M = \begin{bmatrix} 2 & 3/2 & 1/2 & -1/2 \\ 5 & . & . & . \\ 3 & . & . & . \end{bmatrix}$$

<u>STEP 3</u> : Element of 2nd column of D.M

$$l_{22} = a_{22} - l_{21}u_{12} = 1 - 5\left(\frac{3}{2}\right)$$

$$= 1 - \frac{15}{2}$$

$$= -13/2$$

$$u_{32} = a_{32} - l_{31}u_{12}$$

$$= 2 - 3(3/2)$$

$$= -5/2$$

$$D.M = \begin{bmatrix} 2 & 3/2 & 1/2 & -1/2 \\ 5 & -13/2 & \cdot & \cdot \\ 3 & -5/2 & \cdot & \cdot \end{bmatrix}$$

STEP 4 : Elements of 2^{nd} row of D.M

$$u_{23} = \frac{a_{23} - l_{21} u_{13}}{l_{22}} = \frac{1 - 5(1/2)}{-13/2}$$

$$= \frac{2 - 5/2}{-13/2}$$

$$u_{23} = 3/13$$

$$y_2 = \frac{b_2 - l_{21} y_1}{l_{22}} = \frac{9 - 5(-1/2)}{-13/2}$$

$$= \frac{18 + 5/2}{-13/2} = \frac{-23}{13}$$

$$D.M = \begin{bmatrix} 2 & 3/2 & 1/2 & -1/2 \\ 5 & -13/2 & 3/13 & -23/13 \\ 3 & -5/2 & \cdot & \cdot \end{bmatrix}$$

STEP 5 : Elements of 3^{rd} column of D.M

$$l_{33} = a_{33} - l_{31} u_{13} - l_{32} u_{23}$$

$$= 4 - 3(1/2) - (-5/2)(3/13)$$

$$= 4 - 3/2 + \frac{15}{26}$$

$$= 5/2 + \frac{15}{26} = \frac{65}{26} + \frac{15}{26} = \frac{80}{26}$$

$$= \frac{40}{13}$$

$$D.M = \begin{bmatrix} 2 & 3/2 & 1/2 & -1/2 \\ 5 & -13/2 & 3/13 & -23/13 \\ 3 & -5/2 & 40/13 & . \end{bmatrix}$$

<u>STEP 6</u> : Elements of the 3rd row

$$y_3 = \frac{b_3 - (l_{31} y_1 + l_{32} y_2)}{l_{33}}$$

$$= \frac{11 - \left[3\left(-\frac{1}{2}\right) + \left(-\frac{5}{2}\right)\left(-\frac{23}{13}\right)\right]}{40/13}$$

$$= \frac{11 + \frac{3}{2} - \frac{115}{26}}{40/13} = \frac{\frac{286 + 39 - 115}{26}}{40/13}$$

$$= \frac{325}{80} = \frac{21}{8}$$

$$D.M = \begin{bmatrix} 2 & 3/2 & 1/2 & -1/2 \\ 5 & -13/2 & 3/13 & -23/13 \\ 3 & -5/2 & 40/13 & 21/8 \end{bmatrix}$$

The Solution $UX = Y$

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1/2 \\ -23/13 \\ 21/8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 3/13 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1/2 \\ -23/13 \\ 21/8 \end{bmatrix}$$

$$x + \frac{3}{2} y + \frac{1}{2} z = -1/2 \quad -① $$

$$y + \frac{3}{13} z = -23/13 \quad -② $$

$$\boxed{z = 21/8}$$

Sub $z = 21/8$ in ②

$$y + \frac{3}{13}\left(\frac{21}{8}\right) = \frac{-23}{13}$$

$$y + \frac{63}{104} = \frac{-23}{13}$$

$$y = \frac{-23}{13} - \frac{63}{104} = \frac{-184-63}{104}$$

$$y = \frac{-247}{104} \; ; \; \boxed{y = -\frac{19}{8}}$$

Sub $z = 21/8$ and $y = \frac{-19}{8}$ in ①

$$x + \frac{3}{2}\left(\frac{-19}{8}\right) + \frac{1}{2}\left(\frac{21}{8}\right) = -\frac{1}{2}$$

$$x - \frac{57}{16} + \frac{21}{16} = -1/2$$

$$x - \frac{36}{16} = -1/2$$

$$x = \frac{36}{16} - \frac{1}{2} = \frac{72-16}{32} = \frac{56}{32}$$

$$\boxed{x = 7/4}$$

$\therefore$ The values are $x = 7/4$, $y = -19/8$ and

$z = 21/8$

---

3 Solve the equation using crout's method

$$x + y + 2z = 7$$

$$3x + 2y + 4z = 13$$

$$4x + 3y + 2z = 8$$

$$[A, B] \Rightarrow \begin{bmatrix} \overset{a_{11}}{1} & \overset{a_{12}}{1} & \overset{a_{13}}{2} & \overset{b_1}{7} \\ \overset{a_{21}}{3} & \overset{a_{22}}{2} & \overset{a_{23}}{4} & \overset{b_2}{13} \\ \overset{a_{31}}{4} & \overset{a_{32}}{3} & \overset{a_{33}}{2} & \overset{b_3}{8} \end{bmatrix}$$

$$D.M = \begin{bmatrix} l_{11} & u_{12} & u_{13} & y_1 \\ l_{21} & l_{22} & u_{23} & y_2 \\ l_{31} & l_{32} & l_{33} & y_3 \end{bmatrix}$$

STEP 1 : Elements of 1st column of D.M

$$D.M = \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ 3 & \cdot & \cdot & \cdot \\ 4 & \cdot & \cdot & \cdot \end{bmatrix}$$

STEP 2 : Elements of 1st row of D.M

$u_{12} = 1$ $\quad u_{13} = 2$ $\quad y_1 = 7$

$$D.M = \begin{bmatrix} 1 & 1 & 2 & 7 \\ 3 & \cdot & \cdot & \cdot \\ 4 & \cdot & \cdot & \cdot \end{bmatrix}$$

STEP 3 : Elements of 2^{nd} column of D.M

$$l_{22} = a_{22} - l_{21} u_{12} = 2 - 3(1)$$

$$= -1$$

$$l_{32} = a_{32} - l_{31} u_{12} = 3 - 4(1) = -1$$

$$D.M = \begin{bmatrix} 1 & 1 & 2 & 7 \\ 3 & -1 & . & . \\ 4 & -1 & . & . \end{bmatrix}$$

STEP 4 : Elements of 2^{nd} row of D.M

$$u_{23} = \frac{a_{23} - l_{21} u_{13}}{l_{22}} = \frac{4 - 3(2)}{-1}$$

$$= 2$$

$$y_2 = \frac{b_2 - l_{21} y_1}{l_{22}} = \frac{13 - 3(7)}{-1} = 8$$

$$D.M = \begin{bmatrix} 1 & 1 & 2 & 7 \\ 3 & -1 & 2 & 8 \\ 4 & -1 & . & . \end{bmatrix}$$

STEP 5 : Elements of 3^{rd} column of D.M

$$l_{33} = a_{33} - l_{31} u_{13} - l_{32} u_{23}$$

$$= 2 - 4(2) - (-1)(2) = 2 - 8 + 2$$

$$l_{33} = -4$$

$$D.M = \begin{bmatrix} 1 & 1 & 2 & 7 \\ 3 & -1 & 2 & 8 \\ 4 & -1 & -4 & . \end{bmatrix}$$

STEP 6 : Elements of 3^{rd} row of D.M

$$y_3 = \frac{b_3 - (l_{31} y_1 + l_{32} y_2)}{l_{33}}$$

$$= \frac{8 - [4(7) + (-1)(8)]}{-4}$$

$$= \frac{8 - 28 + 8}{-4} = +12/4 = 3$$

$y_3 = 12/4 = 3$

$$D.M = \begin{bmatrix} 1 & 1 & 2 & 7 \\ 3 & -1 & 2 & 8 \\ 4 & -1 & -4 & 12/4 \end{bmatrix}$$

The Solution $UX = Y$

$$\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 12/4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 12/4 \end{bmatrix}$$

$x + y + 2z = 7$ — ①

$y + 2z = 8$ — ②

$\boxed{z = 12/4}$ $\boxed{z = 3}$

Sub $z = 12/4$ in (2)

$$y + 2\left(\frac{12}{4}\right) = 8$$

$$y = 8 - 6$$

$$\boxed{y = +2}$$

$$\begin{array}{r} 18 \\ 11 \\ \hline 7 \end{array}$$

Sub $y = +2$ and $z = 11/4$ in (1)

$$x + (+2) + 2\left(\frac{12}{4}\right) = 7$$

$$x = 7 - \frac{12}{2} - 2$$

$$x = \frac{14 - 12 - 4}{2} = \frac{-2}{2}$$

$$\boxed{x = -1}$$

$$\begin{array}{r} 11 \\ 21 \\ \hline 11 \\ \hline 10 \end{array}$$

∴ The values are $x = -1$, $y = +2$ and $z = 12/4$

The values are $x = -1$, $y = 2$, $z = 3$