MARUDHAR KESARI JAIN COLLEGE FOR WOMEN,VANIYAMBADI
PG & RESEARCH DEPARTMENT OF MATHEMATICS

SUBJECT NAME: MATHEMATICS FOR STATISTICS

CLASS : 1 B.Sc STATISTICS

CODE: 23UEST13

SYLLABUS:

Unit-I Rational fractions: Proper and improper rational fractions. Partial fractions: Forms of partial fractions

UNIT- I

Rational Fractions:

Polynomial:

A function which is the sum of positive integral powers of a variable, say x, is called a polynomial in x.

Example:

Polynomial	Degree
a_0x+a_1	1
$a_0x^2+a_1x+a_2$	2
$a_0x^3+a_1x^2+a_2x+a_3$.	3

Rational fraction:

A function is a form

$$\frac{\text{a Polynomial (or) rational number}}{\text{a Polynomial}}$$ is

called rational fractions.

Proper rational fractions

If the numerator of a rational fraction is of a lower degree than the denominator, then then fraction is called proper rational fraction

Example:

$$\frac{4x+3}{2x^2+3x+1}, \quad \frac{5}{2x^2+3x+1}$$

Improper Rational Fraction:

If the degree of the numerator of a rational fraction is equal to or greater than the degree of the denominator, then the rational function is called improper fraction

Example:

$$\frac{3x^2+4x+3}{2x^2+3x+1}, \quad \frac{7x^3+6x^2+x+2}{2x^2+3x+1}.$$

$$\begin{array}{r|l} & \quad 2x+1 \\ \hline 2x^2+x-1 & 4x^3+4x^2+3x \\ & 4x^3+2x^2-2x \\ & (-)\quad\ (-)\quad\ (+) \\ \hline & \qquad 2x^2-5x+0 \\ & \qquad 2x^2+x-1 \\ \hline & \qquad\qquad 4x+1 \end{array}$$

→ improper to proper

Note:

An improper rational fraction can be expressed as a sum of polynomial and a proper rational fraction

Partial Fraction:

S.no	Factors	Form of Partial fraction's
1.	$x-a$	$\frac{A}{(x-a)}$
2.	x^2+ax+b	$\frac{Ax+B}{x^2+ax+b}$
3.	x^3+ax^2+bx+c	$\frac{Ax^2+Bx+C}{x^3+ax^2+bx+c}$
4.	$(x-a)^2$	$\frac{A}{(x-a)}+\frac{B}{(x-a)^2}$
5.	$(x-a)^3$	$\frac{A}{(x-a)}+\frac{B}{(x-a)^2}+\frac{C}{(x-a)^3}$
6.	$(x^2+ax+b)^2$	$\frac{Ax+B}{x^2+ax+b}+\frac{Cx+D}{(x^2+ax+b)^2}$

1) Split $\frac{1}{(x-1)(x+2)^2}$ into partial fraction

Solution:-

$$\frac{1}{(x-1)(x+2)^2}=\frac{A}{(x-1)}+\frac{B}{(x+2)}+\frac{C}{(x+2)^2} \quad -(A)$$

multi by $(x-1)(x+2)^2$ on b s

$$\frac{(x-1)(x+2)^2}{(x-1)(x+2)^2}=\frac{(x-1)(x+2)^2}{x-1}+\frac{(x-1)(x+2)^2}{x+2}+\frac{(x-1)(x+2)^2}{(x+2)^2}$$

$$1=(x+2)^2+(x-1)(x+2)+(x-1) \quad -(1)$$

$(x-1)=0 \quad , x=1$

$(x+2)=0 \quad , x=-2$

Put $x=1$ in equ ①

$1=(x+2)^2+(x-1)(x+2)+(x-1)$

$1=(1+2)^2+(1-1)(1+2)+(1-1)$ ②

$1=(3)^2+(0)(3)+(0)$

$1=9+0+0$

$1=9A \Rightarrow \boxed{A=\frac{1}{9}}$

$x=-2$ in equ ①

$1=(x+2)^2+(x-1)(x+2)+(x-1)$

$1=(-2+2)^2+(-2-1)(-2+2)+(-2-1)$

$1=(0)^2+(-3)(0)+(-3)$

$1=0+0+\{-3\}$

$1=-3C \Rightarrow \boxed{C=-\frac{1}{3}}$

equating x^2 on both side

$0=A+B$

$0=\frac{1}{9}+B \Rightarrow \boxed{B=-\frac{1}{9}}$

Sub the value A & B & C and in equation A

$$\frac{\frac{1}{9}}{(x-1)} - \frac{\frac{1}{9}}{(x+2)} - \frac{\frac{1}{3}}{(x+2)^2}$$

$$\frac{1}{(x-1)(x+2)^2} = \frac{1/9}{(x-1)} + \frac{-1/9}{(x+2)} + \frac{-1/3}{(x+2)^2}$$

Result:

$$\frac{1}{(x-1)(x+2)^2} = \frac{1/9}{(x-1)} + \frac{-1/9}{(x+2)} + \frac{-1/3}{(x+2)^2}$$

(2) Split $\frac{x+4}{(x^2-4)(x+1)}$ into paratial fraction.

Solution:-

$a^2-b^2=(a+b)(a-b)$

$$\frac{x+4}{(x^2-2^2)(x+1)} = \frac{x+4}{(x+2)(x-2)(x+1)}$$

$$\frac{x+4}{(x+2)(x-2)(x+1)} = \frac{A}{(x+2)} + \frac{B}{(x-2)} + \frac{C}{(x+1)} \quad -(A)$$

multi by $(x+2)(x-2)(x+1)$

$$\frac{(x+4)(x+2)(x-2)(x+1)}{(x+2)(x-2)(x+1)} = \frac{(x+2)(x-2)(x+1)}{(x+2)}$$

$$+ \frac{(x+2)(x-2)(x+1)}{(x-2)} + \frac{(x+2)(x-2)(x+1)}{(x+1)}$$

$$(x+4) = (x-2)(x+1) + (x+2)(x+1) + (x+2)(x-2) \quad -(1)$$

$x+2=0$, $x-2=0$, $x+1=0$

$\boxed{x=-2}$ $\boxed{x=2}$ $\boxed{x=-1}$

Put $x = -2$ equ ①

$$-2+4 = (-2-2)(-2+1) + (-2+2)(-2+1) + (-2+2)(-2-2)$$

$$2 = (-4)(-1) + (0)(-1) + (0)(-4)$$

$$2 = 4A \Rightarrow A = \frac{2}{4} \Rightarrow \boxed{A = 1/2}$$

Put $x = 2$ equ ①

$$2+4 = (2-2)(2+1) + (2+2)(2+1) + (2+2)(2-2)$$

$$6 = (0)(3) + (4)(3) + (4)(0)$$

$$6 = 0 + 12 + 0$$

$$6 = 12B \Rightarrow B = 6/12 \Rightarrow \boxed{B = 1/2}$$

Put $x = -1$ equ ①

$$(-1+4) = (-1-2)(-1+1) + (-1+2)(-1+1) + (-1+2)(-1-2)$$

$$3 = (-3)(0) + (1)(0) + (1)(-3)$$

$$3 = 0 + 0 + (-3)$$

$$3 = -3C \Rightarrow C = -3/3 \Rightarrow \boxed{C = -1}$$

$$\frac{x+4}{(x+2)(x-2)(x+1)} = \frac{1/2}{(x+2)} + \frac{1/2}{(x-2)} + \frac{-1}{(x+1)}$$

Result:-

$$\frac{x+4}{(x+2)(x-2)(x+1)} = \frac{1/2}{(x+2)} + \frac{1/2}{(x-2)} + \frac{-1}{(x+1)}$$

③ Split $\frac{2x^3+3x+4}{(x-1)(x^2+2)}$ into partial fraction

Solution:-

$$\frac{2x^3+3x+4}{(x-1)(x^2+2)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+2)} \longrightarrow (A)$$

$\times$ by $(x-1)(x^2+2)$ on both side

$$\frac{(2x^3+3x+4)(x-1)(x^2+2)}{(x-1)(x^2+2)} = \frac{A(x-1)(x^2+2)}{(x-1)} + \frac{(Bx+C)(x-1)(x^2+2)}{(x^2+2)}$$

$$(2x^3+3x+4) = A(x^2+2)+(Bx+C) \quad — ①$$

$\boxed{x=1}$

$$(2(1)^3+3(1)+4) = A(1^2+2)+(B(1)+C)(1-1)$$

$$(2+3+4) = A(3)+(B+C)(0)$$

$$9 = A(3)$$

$$A = 9/3 \Rightarrow \boxed{A=3}$$

$$2x^3+3x+4 = Ax^2+2A+Bx^2-Bx+Cx-C$$

equating the coefficient of x^2 on b.s

$$0 = A(1)+B(1)$$

$$0 = 3+B$$

$$\boxed{B=-3}$$

equating the coefficient of x on both side

$$3 = -B+C \Rightarrow 3 = 3+C \Rightarrow 3-3 = C \Rightarrow \boxed{C=0}$$

$$\frac{2x^3+3x+4}{(x-1)(x^2+2)} = \frac{3}{(x-1)} + \frac{-3x+0}{(x^2+2)}$$

Result:

$$\frac{2x^3+3x+4}{(x-1)(x^2+2} = \frac{3}{(x-1)} + \frac{-3x+0}{(x^2+2)}$$

4. Split $\frac{3}{(x+1)(x-1)}$ into partial fraction. 5

Solution:-

$$\frac{3}{(x+1)(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} \quad — A$$

$\times$ by $(x+1)(x-1)$

$$\frac{3(x+1)(x-1)}{(x+1)(x-1)} = \frac{A(x+1)(x-1)}{(x+1)} + \frac{B(x+1)(x-1)}{(x-1)}$$

$$3 = A(x-1) + B(x+1) \quad — 1$$

$\boxed{x=-1}$

$$3 = (-1-1) + (-1+1)$$

$$3 = (-2) + 0$$

$$3 = -2A \Rightarrow \boxed{A = -3/2}$$

$\boxed{x=1}$

$$3 = (1-1) + (1+1)$$

$$3 = 0 + 2$$

$$3 = 2B \Rightarrow \boxed{B = 3/2}$$

$$\frac{3}{(x+1)(x-1)} = \frac{\cancel{A}\ -3/2}{(x+1)} + \frac{3/2}{(x-1)}$$

$$\frac{3}{(x+1)(x-1)} = \frac{-3/2}{(x+1)} + \frac{3/2}{(x-1)}$$

(5) Split $\frac{x^4+2x+4}{(x-1)(x^2+1)^2}$ into partial fraction.

Solution:-

$$\frac{x^4+2x+4}{(x-1)(x^2+1)^2} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)} + \frac{Dx+E}{(x^2+1)^2} \quad \text{—— (A)}$$

$\times$ by $(x-1)(x^2+1)^2$ on b.s

$$\frac{(x^4+2x+4)(x-1)(x^2+1)^2}{(x-1)(x^2+1)^2} = \frac{A\cancel{(x-1)}(x^2+1)^2}{\cancel{(x-1)}} + \frac{(Bx+C)(x-1)(x^2+1)^2}{\cancel{(x^2+1)}} + \frac{(Dx+E)(x-1)(x^2+1)^2}{(x^2+1)^2}$$

$(x^2+1)^2 =$

x^4+1

$a \quad b$

$(a+b)^2 = a^2+b^2+2ab$

$$(x^4+2x+4) = A(x^2+1)^2 + (Bx+C)(x-1)(x^2+1) + (Dx+E)(x-1)$$

$A(x^4+1+2x^2)$ $\quad x^3+x-x^2=1$

$\boxed{x=1}$

$$(1^4+2(1)+4) = A(1^2+1)^2 + (B(1)+C)(1-1)(1^2+1) + (B(1)+E)(1-1)$$

$$1+2+4 = A(2)^2+0+0$$

$$7 = A4$$

$$\boxed{A = 7/4}$$

$$x^4+2x+4 = x^4+1+2x^2+Bx^4+Bx^2-Bx^3-Bx+Cx^3+Cx-Cx^2$$
$$+C+Dx^2-Dx+Ex-E$$

co-eff of x^4

$1 = 1A + 1B$

$1 = 1\left(\frac{7}{4}\right) + B$

$1 - \frac{7}{4} = B$

$\frac{4-7}{4} = B$

$-3/4 = B$

$\boxed{B = -3/4}$

co-eff of x^3

$0 = -1B + C$

$0 = -1(-3/4) + C$

$0 = 3/4 + C$

$\boxed{C = -3/4}$

co-eff of x^2

$0 = 2(A) + B(1) - C(1) + D(1)$

$0 = 2A + (-3/4) - (-3/4) + D$

$0 = 2(7/4) - 3/4 + 3/4 + D$

$0 = \frac{14}{4} + D$

$0 = \frac{7}{2} + D$

$-D = \frac{7}{2}$

$\boxed{D = -7/2}$

co-eff of x

$2 = -B + C - D + E$

$2 = -\left(\frac{3}{4}\right) + \frac{-3}{4} - \left(-\frac{7}{2}\right) + E$

$2 = \frac{7}{2} + E$

$2 - \frac{7}{2} = +E$

$\frac{4-7}{2} = E$

$\boxed{E = -3/2}$

$$\frac{x^4+2x+4}{(x-1)(x^2+1)^2} = \frac{7/4}{(x-1)} + \frac{-3/4\,x + -3/4}{(x^2+1)} + \frac{-7/2\,x + -3/2}{(x^2+1)^2}$$

Result:-

$$\frac{x^4+2x+4}{(x-1)(x^2+1)^2} = \frac{7/4}{(x-1)} + \frac{-3/4\,x + -3/4}{(x^2+1)} + \frac{-7/2\,x + -3/2}{(x^2+1)^2}$$

⑥ H/w

Split $\frac{x^2+x+1}{(x-1)(x-2)(x-3)}$ into partial fraction.

Solution:-

$$\frac{x^2+x+1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$\times$ by $(x-1)(x-2)(x-3)$

$$x^2+x+1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$$

$x-1=0$

$x=1$

$$1^2+1+1 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-1)(1-2)$$

$$3 = A(-1)(-2) + B(0)(-2) + C(0)(-1)$$

$$3 = 2A + 0 + 0$$

$$\boxed{\frac{3}{2} = A}$$

$x = 2$

$2^2 + 2 + 1 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-1)(2-2)$

$4 + 3 = A(0)(-1) + B(1)(-1) + C(1)(0)$

$7 = 0 + B(-1) + 0C$

$7 = -1B$

$\boxed{B = -7}$

$x = 3$

$3^2 + 3 + 1 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$

$9 + 4 = A(1)(0) + B(2)(0) + C(2)(1)$

$13 = 0 + 0 + 2C$

$13 = 2C$

$\boxed{\frac{13}{2} = C}$

A, B, C sub in equ 1

$$\frac{x^2 + x + 1}{(x-1)(x-2)(x-3)} = \frac{\frac{3}{2}}{(x-1)} + \frac{-7}{(x-2)} + \frac{\frac{13}{2}}{(x-3)}$$

Result

$$\frac{x^2 + x + 1}{(x-1)(x-2)(x-3)} = \frac{3/2}{(x-1)} + \frac{-7}{(x-2)} + \frac{13/2}{(x-3)}$$

⑦ Split $\frac{7x+4}{(1+x)^2(3x+2)}$ into partial fraction.

Solution:-

$$\frac{7x+4}{(1+x)^2(3x+2)} = \frac{A}{(1+x)} + \frac{B}{\cancel{x}(1+x)^2} + \frac{C}{(3x+2)}$$

$\times$ by $(1+x)^2(3x+2)$

$$7x+4 = A(1+x)(3x+2) + B(3x+2) + C(1+x)^2$$

$x = -1$

$$7(-1)+4 = A(1+(-1))(3(-1)+2) + B(3(-1)+2) + C(1+(-1))^2$$

$$-7+4 = A(0)(-1) + B(-1) + C(0)^2$$

$$-3 = -B$$

$$\boxed{B=3}$$

$$7x+4 = A(3x+2+3x^2+2x) + B3x + 2B + C + Cx^2 + C2x$$

$$7x+4 = A3x + A2 + A3x^2 + A2x + B3x + 2B + C + Cx^2 + C2x$$

co-eff of x^2 on b.s

$0 = 3A + 1C$

$-3A = C \Rightarrow \boxed{C=-3A}$

Co-eff of x on b.s

$7 = 3A + 2A + 3B + 2C$

$7 = 5A + 3B + 2C$

$7 = 5A + 3(3) + 2(-3A)$

$7 = 5A + 9 - 6A$

$7 = -A + 9$

$7 - 9 = -A$

$-2 = -A$

$\boxed{A=2}$

Co-eff constant

$$4 = 2A + 2B + C$$

$$4 = 2(2) + 2(3) + C$$

$$4 = 4 + 6 + c$$

$$4 = 10 + c$$

$$4 - 10 = c$$

$$-6 = c$$

$$\boxed{C = -6}$$

⑧ Split $\dfrac{x^2-10x+13}{(x-1)(x^2-5x+6)}$ into partial fraction

Solution:-

$$\frac{x^2-10x+13}{(x-1)(x^2-5x+6)} = \frac{x^2-10x+3}{(x-1)(x-2)(x-3)}$$

$$\frac{x^2-10x+13}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$\times$ by $(x-1)(x-2)(x-3)$ on b.s

$$x^2-10x+13 = A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2) \quad -①$$

$x=1$

$$1^2-10(1)+13 = A(1-2)(1-3)+B(1-1)(1-3)+C(1-1)(1-2)$$

$$1-10+13 = A(-1)(-2)+B(0)(-2)+C(0)(-1)$$

$$-10+14 = 2A+0+0$$

$$-4 = 2A$$

$$A = 4/2$$

$$\boxed{A=2}$$

$x=2$

$$2^2-10(2)+13 = A(2-2)(2-3)+B(2-1)(2-3)+C(2-1)(2-2)$$

$$4-20+13 = 0+B(1)(-1)+0$$

$$-20+17 = -B$$

$$-3 = -B$$

$$\boxed{B=3}$$

$x = 3$

$$3^2 - 10(3) + 13 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-1)(3-2)$$

$$9 - 30 + 13 = 0 + 0 + C(2)(1)$$

$$-30 + 22 = 2C$$

$$-8 = 2C$$

$$C = -8/2$$

$$\boxed{C = -4}$$

$$\frac{x^2 - 10x + 13}{(x-1)(x-2)(x-3)} = \frac{2}{(x-1)} + \frac{3}{(x-2)} + \frac{-4}{(x-3)}$$

Result:

$$\frac{x^2 - 10x + 13}{(x-1)(x-2)(x-3)} = \frac{2}{(x-1)} + \frac{3}{(x-2)} + \frac{-4}{(x-3)}$$

H/W

⑨ Split $\frac{2x-3}{(x-2)(x+1)^2}$ into partial fraction.

Solution :-

$$\frac{2x-3}{(x-2)(x+1)^2} = \frac{A}{(x-2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$\times$ by $(x-2)(x+1)^2$

$$2x-3 = A(x+1)^2 + B(x-2)(x+1) + C(x-2)$$

$x = 2$

$$2(2)-3 = A(2+1)^2 + B(2-2)(2+1) + C(2-2)$$

$$4-3 = A(3)^2 + B(0)(3) + C(0)$$

$$1 = 9A + 0 + 0$$

$$1 = 9A$$

$$\boxed{\frac{1}{9} = A}$$

$x = -1$

$$2(-1)-3 = A((-1)+1)^2 + B(-1-2)(-1+1) + C(-1-2)$$

$$-2-3 = A(0)^2 + B(-3)(0) + C(-3)$$

$$-5 = 0 + 0 + C(-3)$$

$$-5 = -3C$$

$$\boxed{C = \frac{3}{5}}$$

$$2x-3 = Ax^2+A+A2x+Bx^2+Bx-2xB-2B$$
$$+Cx-2C$$

Co-eff of x^2 on b.s

$$0 = 1A+1B$$
$$0 = \frac{1}{9}+B$$
$$\boxed{-\frac{1}{9} = B}$$

$$\frac{2x-3}{(x-2)(x+1)^2} = \frac{\frac{1}{9}}{(x-2)} + \frac{-1/9}{(x+1)} + \frac{5/3}{(x+1)^2}$$

Result :-

$$\frac{2x-3}{(x-2)(x+1)^2} = \frac{1/9}{(x-2)} + \frac{-1/9}{(x+1)} + \frac{5/3}{(x+1)^2}$$

⑩ Find the constants A, B, C

$$\frac{x^2-5x+1}{(x+1)(x+2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+1)(x+2)} + \frac{C}{(x+1)(x+2)(x+3)}$$

Solution:-

$$\frac{x^2-5x+1}{(x+1)(x+2)(x+3)} = \frac{A}{(x+1)} + \frac{B}{(x+1)(x+2)} + \frac{C}{(x+1)(x+2)(x+3)}$$

$\times$ by $(x+1)(x+2)(x+3)$

$$x^2-5x+1 = A(x+2)(x+3)+B(x+3)+C$$

$x=-3$

$$(-3)^2-5(-3)+1 = A((-3)+2)((-3)+3)+B(-3+3)+C$$

$$9+15+1 = A(-1)(0)+B(0)+C$$

$$10+15 = C$$

$$\boxed{25 = C}$$

$$x^2-5x+1 = A(x^2+3x+2x+6)+Bx+3B+C$$

$$x^2-5x+1 = Ax^2+3xA+2xA+6A+Bx+3B+C$$

Co. eff of x^2 on b.s

$$1 = 1A$$

$$\boxed{A=1}$$

Co. eff of x on b.s

$$-5 = 3A + 2A + 1B + \cancel{3B}$$

$$-5 = 5A + 1B$$

$$-5 = 5(1) + B$$

$$-5 = 5 + B$$

$$-5 - 5 = B$$

$$\boxed{-10 = B}$$

$$\frac{x^2 - 5x + 1}{(x+1)(x+2)(x+3)} = \frac{1}{(x+1)} + \frac{-10}{(x+1)(x+2)} + \frac{25}{(x+1)(x+2)(x+3)}$$

Result:-

$$\frac{x^2 - 5x + 1}{(x+1)(x+2)(x+3)} = \frac{1}{(x+1)} + \frac{-10}{(x+1)(x+2)} + \frac{25}{(x+1)(x+2)(x+3)}$$

(11) $\dfrac{3x+7}{x^2-3x+2}$ into partial fraction.

Solution:-

$$\frac{3x+7}{x^2-3x+2} = \frac{3x+7}{(x-1)(x-2)}$$

-3, 2, -1, -2

$$\frac{3x+7}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$$

$\times$ by $(x-1)(x-2)$

$$3x+7 = A(x-2)+B(x-1)$$

$x=1$

$$3(1)+7 = A(1-2)+B(1-1)$$

$$3+7 = A(-1)+0$$

$$10 = -A$$

$$\boxed{A=-10}$$

$x=2$

$$3(2)+7 = A(2-2)+B(2-1)$$

$$6+7 = 0+B(1)$$

$$\boxed{13=B}$$

Result:

$$\frac{3x+7}{(x-1)(x-2)} = \frac{-10}{(x-1)} + \frac{13}{(x-2)}$$

(12) $\frac{4x^2-3x+5}{(2-x)(1+x^2)}$ into partial fraction

Solution:-

$$\frac{4x^2-3x+5}{(2-x)(1+x^2)} = \frac{A}{(2-x)} + \frac{Bx+C}{(1+x^2)}$$

$\times$ by $(2-x)(1+x^2)$

$$4x^2-3x+5 = A(1+x^2)+(Bx+C)(2-x)$$

$x=2$

$$4(2)^2-3(2)+5 = A(1+(2)^2)+(B(2)+C)(2-2)$$

$$16-6+5 = 5A+0$$

$$21-6 = 5A$$

$$15 = 5A$$

$$\frac{15}{5} = A \Rightarrow \boxed{A=3}$$

~~co-eff of x^2~~

$$4x^2-3x+5 = A+Ax^2+B2x-Bx^2+2C-Cx$$

co-eff of x^2 on b.s

$$4 = 1A-1B$$

$$4 = 3-B \Rightarrow 4-3 = -B$$

$$\cancel{B=-4+3} \Rightarrow B=1 \quad \boxed{B=1}$$

co-eff of x on b.s

$-3 = 2B - 1C$

$-3 = 2(1) - C$

$-3 = 2 - C \Rightarrow -3 - 2 = -C$

~~co-eff of x on B.S~~

$-5 = -C$

$\boxed{C = 5}$

-3

(or)

co-eff of constant on. b.s

$5 = A + 2C$

$5 = 3 + 2C$

$5 = 5C$

$\frac{5}{5} = C$

$\boxed{C = 1}$

(13) ~~P.T~~ $\frac{1}{(1-ax)^2(1-bx)} = \frac{A}{(1-ax)} + \frac{B}{(1-bx)}$

P.T $\frac{1}{(1-ax)^2(1-bx)} = \frac{A}{(1-ax)^2} + \frac{AB}{(1-ax)} + \frac{B^2}{(1-bx)}$

Solution:-

LHS $\Rightarrow$ $\frac{1}{(1-ax)^2(1-bx)} = \frac{1}{(1-ax)(1-ax)(1-bx)}$

$$\frac{1}{(1-ax)^2(1-bx)} = \frac{1}{(1-ax)}\left[\frac{1}{(1-ax)(1-bx)}\right]$$

By conditions

$$\frac{1}{(1-ax)^2(1-bx)} = \frac{1}{(1-ax)}\left[\frac{A}{(1-ax)} + \frac{B}{(1-bx)}\right]$$

$$= \frac{A}{(1-ax)^2} + \frac{B}{(1-ax)(1-bx)}$$

$$= \frac{A}{(1-ax)^2} + B\left[\frac{1}{(1-ax)(1-bx)}\right]$$

$$= \frac{A}{(1-ax)^2} + B\left[\frac{A}{(1-ax)} + \frac{B}{(1-bx)}\right]$$

$$= \frac{A}{(1-ax)^2} + \frac{BA}{(1-ax)} + \frac{B^2}{(1-bx)}$$

= RHS //-

(14) Split $\frac{3x+1}{(3x+4)^2}$ into partial fractions.

Solution:-

$$\frac{3x+1}{(3x+4)^2} = \frac{A}{3x+4} + \frac{B}{(3x+4)^2}$$

$\times$ by $(3x+4)^2$

$$3x+1 = A(3x+4) + B$$

$$3x+1 = A3x + A4 + B$$

Co-eff of x

$$3 = 3A$$

$$\frac{3}{3} = A$$

$$\boxed{A = 1}$$

Co-eff of constant

$$1 = A4 + B$$

$$1 = 1(4) + B$$

$$1 = 4 + B$$

$$1 - 4 = B$$

$$-3 = B$$

$$\boxed{B = -3}$$

Result:

$$\frac{3x+1}{(3x+4)^2} = \frac{1}{3x+4} + \frac{-3}{(3x+4)^2}$$

(15) Split $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$ into partial fractions.

Solution:-

$$\frac{2x^2+3x+4}{(x-1)(x^2+2)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+2)}$$

$\times$ by $(x-1)(x^2+2)$

$$\frac{2x^2+3x+4}{\cancel{(x-1)(x^2+2)}} = A(x^2+2)+(Bx+C)(x-1)$$

$x = 1$

$$2(1)^2+3(1)+4 = A((1)^2+2)+B(1)+C(\cancel{x}-1)$$

$$2+3+4 = 3A+0$$

$$9 = 3A$$

$$\frac{9}{3} = A$$

$$\boxed{A = 3}$$

~~equ of coeff of x^2~~

$$2x^2+3x+4 = Ax^2+2A+Bx^2-Bx+Cx-C$$

co-eff of x^2

$$2 = A+B$$

$$2 = 3+B$$

$$2-3 = B$$

$$-1 = B$$

$$\boxed{B = -1}$$

Co-eff of x

$$3 = -B + C$$

$$3 = 1 + C$$

$$3 - 1 = C$$

$$2 = C$$

$$\boxed{C = 2}$$

Result:

$$\frac{2x^2 + 3x + 4}{(x-1)(x^2+2)} = \frac{3}{(x-1)} + \frac{-1x + 2}{(x^2+2)}$$

$$\frac{2x^2 + 3x + 4}{(x-1)(x^2+2)} = \frac{3}{(x-1)} + \frac{2 - x}{(x^2+2)}$$