MARUDHAR KESARI JAIN COLLEGE FOR WOMEN,VANIYAMBADI
PG & RESEARCH DEPARTMENT OF MATHEMATICS

SUBJECT NAME: MATHEMATICS FOR STATISTICS

CLASS : 1 B.Sc STATISTICS

CODE: 23UEST13

SYLLABUS:

Unit-III Theory of equations: Polynomial equations with real coefficients- imaginary and irrational roots-solving equations with related roots-equation with given numbers as roots.

Theory of Equations

Polynomial Equation with real co-efficient.

Consider the polynomial equation

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n = 0$$

where,

$a_0, a_1, a_2, \cdots a_n$ real co-efficient. its degree is n.

and it has n roots if $\alpha_1, \alpha_2, \alpha_3 \cdots \alpha_n$ are the roots then we have

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_n$$

$$= a_0(x-\alpha_1)(x-\alpha_2)\cdots(x-\alpha_n) \quad -(1)$$

$$= a_0[x^n - (\alpha_1 + \alpha_2 + \cdots \alpha_n)x^{n-1} + (-1)^n(\alpha_1\alpha_2\cdots\alpha_n)]$$

$$= a_0[x^n - S_1x^{n-1} + S_2x^{n-2} + \cdots + (-1)S_n] \quad -(2)$$

Where, S_1 = Sum of roots $= \alpha_1 + \alpha_2 + \cdots + \alpha_n$

S_2 = Sum of the products of the roots taken to 2 at a time

$$= (\alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_1\alpha_4 + \cdots +) + (\alpha_2\alpha_3 + \alpha_3\alpha_4$$

$\vdots$

S_n = product of all the n roots $= (\alpha_1\alpha_2\alpha_3 \cdots$

Comparing the coefficient $x^{n-1}, x^{n-2} \ldots x$ and the constant term into in 2

$$a_1 = -a_0 S_1$$
$$a_2 = -a_0 S_2$$
$$a_3 = -a_0 S_3$$
$$\vdots$$
$$a_n = (-1)^n a_0 S_n$$

$$\therefore S_1 = \frac{-a_1}{a_0} = -\frac{\text{co-eff of } x^{n-1}}{\text{co-eff of } x^n}$$

$$S_2 = \frac{a_2}{a_0} = \frac{\text{co-eff of } x^{n-2}}{\text{co-eff of } x^n}$$

$$\vdots$$

$$S_n = (-1)^n \frac{a_n}{a_0} = \frac{(-1)^n \text{ constant term}}{\text{coff of } x^n}$$

$$(-1)^3 \frac{6}{2} = -3$$

If α, β, γ are the roots of $2x^3+3x^2+5x+6=0$ $\Sigma\alpha$, $\Sigma\beta$ and $\alpha\beta\gamma$ find.

Solution:-

$$\Sigma\alpha = \alpha+\beta+\gamma$$

$$S_1 = -\frac{a_1}{a_0} = \frac{\text{co-eff of } x^{n-1}}{\text{co-eff of } x^n}$$

Given: $2x^3+3x^2+5x+6=0$

$$S_1 = -\frac{\text{co eff of } x^2}{\text{co-eff of } x^3} = -\frac{3}{2}$$

$\Sigma \alpha\beta = \alpha\beta + \beta\gamma + \gamma\alpha$

$$S_2 = \frac{a_2}{a_0} = \frac{\text{coff of } x^{n-1}}{\text{coff of } x^n}$$

$$S_2 = \frac{5}{2}$$

$\alpha\beta\gamma = \alpha\beta\gamma$

$$S_3 = \cancel{-\frac{a_3}{a_0}} = (-1)^3 \frac{6}{2}$$

$$= -3 //-$$

Remark.

A quardratic equation is of the form $x^2-(\alpha+\beta)x+\alpha\beta$

Sum of the roots $= \alpha + \beta$

products of the root $= \alpha\beta$

② Solve $x^4+2x^3-5x^2+6x+2=0$ given that $1+i$ is a root

solution

Given that $1+i$ is a root and its conjugate $1-i$ is also a root.

The equation has 4 roots

Let the other two roots α, β

Sum of roots

$$(1+i)+(1-i)+\alpha+\beta = \frac{\text{coff of } x^3}{\text{coff of } x^4}$$

$$1+i+1-i+\alpha+\beta=-\frac{2}{1}$$

$$2+\alpha+\beta=-2$$

$$\alpha+\beta=-2-2$$

$$\boxed{\alpha+\beta=-4}$$

product of a roots

$$(1+i)(1-i)\alpha\beta=\frac{(-1)^4\text{ co.eff of constant term}}{\text{co.eff of }x^4}$$

$$(1^2-i^2)\alpha\beta=\frac{2}{1}$$

$$(1+1)\alpha\beta=2$$

$$2\alpha\beta=2$$

$$\alpha\beta=2/2$$

$$\boxed{\alpha\beta=1}$$

$$x^2+(-4x)+1=0$$

$$x^2-(\alpha+\beta)x+\alpha\beta=0$$

$$x^2+4x+1=0$$

$a=1,\ b=4,\ c=1$

$$=\frac{-b\pm\sqrt{b^2-4ac}}{4a}$$

$$=\frac{-4\pm\sqrt{(4)^2-4(1)(1)}}{4(1)}$$

$$=\frac{-4\pm\sqrt{16-4}}{4}$$

$$= \frac{-4 \pm \sqrt{12}}{2}$$

$$= \frac{-4 \pm \sqrt{4 \times 3}}{2}$$

$$= \frac{-4 \pm 2\sqrt{3}}{2}$$

$$= \frac{2(-2 \pm \sqrt{3})}{2}$$

$$= -2 \pm \sqrt{3}$$

$x = -2 + \sqrt{3}$, $x = -2 - \sqrt{3}$

The four roots are

$1+i, \; 1-i, \; -2+\sqrt{3}, \; -2-\sqrt{3}$

3) Solve $x^4 - 11x^2 + 2x + 12 = 0$ given that $\sqrt{5}-1$ is a root

Solution:-

Given that $\sqrt{5}-1$ is a root & its conjugate $-\sqrt{5}-1$ is also a root

This equation 4 roots

let other two roots α, β

Sum of the root

$$(\sqrt{5}-1) + (-\sqrt{5}-1) + \alpha + \beta = \frac{\text{coff of } x^3}{\text{coff of } x^4}$$

$\sqrt{5}+1-\sqrt{5}-1+\alpha+\beta=\frac{0}{1}$

$-2+\alpha+\beta=0$

$\boxed{\alpha+\beta=2}$

product of a roots

$(\sqrt{5}-1)(-\sqrt{5}-1)\alpha\beta=\frac{(-1)^{4}\text{coeff of } x^{0} \text{ constant term}}{\text{coeff of } x^{4}}$

$(-\sqrt{5})^{2}-$

$((-\sqrt{5})^{2}-1^{2})\alpha\beta=-\frac{12}{1}$

$(-5+1)\alpha\beta=-12$

$-4\alpha\beta=-12$

$\alpha\beta=-12/-4$

$\boxed{\alpha\beta=-3}$

$x^{2}-(\alpha+\beta)x+\alpha\beta=0$

$x^{2}-2x+(-3)=0$

$x^{2}-2x-3=0$
Add multiple

$(x-3)(x+1)=0$

$x-3=0 \quad x+1=0$

$x=3 \quad x=-1$

The four roots are

$\sqrt{5}-1, -\sqrt{5}-1, 3, -1$

$\sqrt{5} - 1 - \sqrt{5} - 1 + \alpha + \beta = -\frac{0}{1}$

$-2 + \alpha + \beta = 0$

$\boxed{\alpha + \beta = 2}$

Product of a roots

$(\sqrt{5} - 1)(-\sqrt{5} - 1)\alpha\beta = \frac{(-1)^4 \text{ coeff of } x^0 \text{ constant term}}{\text{coeff of } x^4}$

$\cancel{(-\sqrt{5})^2 -}$

$((-\sqrt{5})^2 - 1^2)\alpha\beta = -\frac{12}{1}$

$(-5 + 1)\alpha\beta = -12$

$-4\alpha\beta = 12$

$\alpha\beta = 12/-4$

$\boxed{\alpha\beta = -3}$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$x^2 - 2x + (-3) = 0$

$x^2 - 2x - 3 = 0$

Add multiple

$(x - 3)(x + 1) = 0$

$x - 3 = 0 \qquad x + 1 = 0$

$x = 3 \qquad x = -1$

The four roots are

$\sqrt{5} - 1, -\sqrt{5} - 1, 3, -1$

(4) $x^4+4x^3+5x^2+2x-2=0$ given that $-1+i$ is a root

Solution:

Given that $-1+i$ is a root and its conjugat $-1-i$ is also root

This equation has 4 roots

let other two roots α, β

sum of the roots

$$(-1+i)+(-1-i)+\alpha+\beta = \frac{\text{coeff of } x^3}{\text{coeff of } x^4}$$

$$-1+i-1-i+\alpha+\beta = -\frac{4}{1}$$

$$-2+\alpha+\beta = -4$$

$$\alpha+\beta = -4+2$$

$$\boxed{\alpha+\beta = -2}$$

product of a root

$$(-1+i)(-1-i)\alpha\beta = \frac{(-1)^4 \text{ co-eff of constant term}}{\text{co-eff of } x^4}$$

$$(-1^2-i^2)\alpha\beta = \frac{-2}{1}$$

$$(1+1)\alpha\beta = -2$$

$$2\alpha\beta = -2$$

$$\alpha\beta = -2/2$$

$$\boxed{\alpha\beta = -1}$$

$$x^2 \cancel{+} - (\alpha+\beta)x + \alpha\beta = 0$$

$$x^2 + 2x - 1 = 0$$

$a = 1, b = 2, c = -1$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4+4}}{2}$$

$$= \frac{-2 \pm \sqrt{8}}{2}$$

$$= \frac{-2 \pm \sqrt{2 \times 4}}{2}$$

$$= \frac{-2 \pm 2\sqrt{2}}{2} \qquad \frac{\cancel{2}(-1 \pm \sqrt{2})}{\cancel{2}}$$

$$= -1 \pm \sqrt{2}$$

$$= -1 + \sqrt{2} \qquad = -1 - \sqrt{2}$$

(5) $x^4 - 4x^2 + 8x + 35 = 0$ given that $2 + i\sqrt{3}$ is a root.

9. Solve $3x^4 - 4x^4 - 43x^3 + 50x^2 + 27x - 36 = 0$ if $\sqrt{2} - \sqrt{5}$ is a root.

Solution:-

Given that $\sqrt{2} - \sqrt{5}$ is a root other roots are

$\sqrt{2} + \sqrt{5}$, $-\sqrt{2} - \sqrt{5}$, $-\sqrt{2} + \sqrt{5}$, α.

$$= \frac{\text{Coeff of } x^{n-1}}{\text{Coeff of } x^{n}}$$

Sum of the roots

$$\sqrt{2} - \sqrt{5} + \sqrt{2} + \sqrt{5} - \sqrt{2} - \sqrt{5} - \sqrt{2} + \sqrt{5} + \alpha = -\frac{x^4}{x^5}$$

$$\alpha = \frac{-(-4)}{3}$$

$$\boxed{\alpha = \frac{4}{3}}$$

10 Solve $x^4 + 4x^3 + 2x^2 - 12x + 9 = 0$ given that it has two pairs of equal roots.

Solution:-

Two pairs of equal roots are $\alpha, \alpha, \beta, \beta$

$$\alpha + \alpha + \beta + \beta = \frac{\text{coeff of } x^3}{\text{coeff of } x^4}$$

$$2\alpha + 2\beta = -\frac{4}{1}$$

$$2(\alpha + \beta) = -4 \Rightarrow$$

product of the root $= (-1)^4 \dfrac{\text{coeffecient of constant term}}{\text{coeff of } x^4}$

$$(\alpha)(\alpha)(\beta)(\beta) = (-1)\frac{9}{1}$$

$$(\alpha)^2(\beta)^2 = 9$$

$$(\alpha\beta)^2 = 9$$

$$\alpha\beta = \sqrt{9}$$

$$\boxed{\alpha\beta = \pm 3}$$

$$x^2 - (\alpha+\beta)x + \alpha\beta = 0$$

$$\cancel{x^2 - 3x + \alpha\beta}$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$\boxed{x=-3} \quad \boxed{x=1}$$

-3, -3, $+1$

$x^2 + 2x - 3 = 0$ $a=1, b=2, c=-3$

$$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)}$$

The four roots are $-3, 1, 3, 1$

$$= \frac{-2 \pm \sqrt{4+12}}{2}$$

$$= \frac{-2 \pm \sqrt{16}}{2}$$

$$= \frac{-2 \pm 4}{2} \Rightarrow \frac{2(-1 \pm 2)}{2} = -1 \pm 2, \ -1-2 = 1, -3$$

$x^2+2x+3=0$ $\qquad a=1,\ b=2,\ c=3$

$$=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-2\pm\sqrt{2^2-4(1)(3)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{4-12}}{2}$$

$$=\frac{-2\pm\sqrt{-8}}{2}$$

Solving equation with related roots:

(1) If the roots of $x^3+px^2+qx+r=0$ are in a.p

S.T $2p^3-9pq+27r=0$

Solution:-

Let the roots be $a-d, a, a+d$

$$S_1=-\frac{a_1}{a_0}$$

$$a-d+a+a+d=\frac{-p}{1}$$

$$3a=-p$$

$$\boxed{a=-p/3}$$

$$S_2=\frac{a_2}{a_0}$$

$$(a-d)a+a(a+d)+(a+d)(a-d)=\frac{q}{1}$$

$$a^2-ad+a^2+ad+a^2-ad+ad-d^2=q$$

$$3a^2-d^2=q$$

$$3(-p/3)^2-d^2=q$$

$$\frac{3p^2}{9} - d^2 = q$$

$$\frac{p^2}{3} - d^2 = q \qquad -d^2 = q - \frac{p^2}{3} \Rightarrow d^2 = -q + \frac{p^2}{3}$$

$$d^2 = \frac{p^2}{3} - q \quad -② $$

$$S_3 = (-1)^3 \frac{a_3}{a_0}$$

$$= -\frac{r}{1}$$

$$(a-d)a(a+d) = -r$$

$$(a-d)(a^2+ad) = -r$$

$$a^3 - a^2d + a^2d - ad^2 = -r$$

$$a^3 - ad^2 = -r$$

$$-ad^2 = -r - a^3 \Rightarrow -(ad^2) = -(r+a^3) \Rightarrow ad^2 = r + a^3$$

$$d^2 = \frac{r+a^3}{a}$$

$$d^2 = \frac{r}{a} + \frac{a^3}{a}$$

$$d^2 = \frac{r}{a} + a^2$$

$$d^2 = \frac{r}{(-p/3)} + (-p/3)^2$$

$$d^2 = -\frac{3r}{p} + \frac{p^2}{9} \quad -③$$

② = ③

$$\cancel{a_6 + \frac{p^2}{3}} \quad -q + \frac{p^2}{3} = \frac{-3r}{p} + \frac{p^2}{9}$$

$$\frac{-3q + p^2}{3} = \frac{p^2}{9} - \frac{3r}{p}$$

$$\frac{-3q + p^2}{3} = \frac{p^3 - 27r}{9p}$$

$$p^2 - 3q = \frac{p^3 - 27r}{3p}$$

$$3p(p^2 - 3q) = p^3 - 27r$$

$$3p^3 - 9qp = p^3 - 27r$$

$$3p^3 - 9pq - p^3 + 27r = 0$$

$$2p^3 - 9pq + 27r = 0.$$

(2) Solve the following equation given that its roots are A.P $x^3 - 12x^2 + 39x - 28 = 0$

Solution:-

Since the roots are in A.P

Let $a-d, a, a+d$

$$S_1 = \frac{a_1}{a_0}$$

$$a - d + a + a + d = \frac{-(-12)}{1}$$

$$3a = 12 \Rightarrow \boxed{a = 4}$$

$$S_3 = (-1)^3 \frac{a_3}{a_0}$$

$$(a-d)(a)(a+d) = \frac{-(-28)}{1}$$

$$a^3 - ad^2 = 28$$

$$(4)^3 - (4)d^2 = 28$$

$$64 - 4d^2 = 28$$

$$-4d^2 = 28 - 64$$

$$-4d^2 = -36$$

$$d^2 = 9 \Rightarrow d = \pm 3$$

$a = 4$, $d = 3$, $d = -3$

$(a-d, a, a+d) = (4-3, 4, 4+3)$

$= (1, 4, 7)$

$a = 4$, $P = -3$

$a = (4-3, 4, 4+3)$

$= (1, 4, 7)$

(4) Solve $6x^3 - 11x^2 - 3x + 2 = 0$ given that it roots are in H.P

Solution:

If a, b, c are h.p then the reciprocals $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$

∴ the equation those roots are the reciprocal of roots of the given function equation is $2y^3 - 3y^2 - 11y + 6 = 0$

(This is obtain from the given equation by reversing its co.eff)

Therefore, the roots of the equation are in A.P.

Let the roots are $a-d, a, a+d$

$$S_1 = \frac{a_1}{a_0}$$

$$a - d + a + a + d = \cancel{\tfrac{6}{6}} - \frac{(-3)}{2}$$

$$3a = \frac{3}{2}$$

$$a = \frac{3}{3 \times 2} \Rightarrow a = \frac{3}{6} \Rightarrow \boxed{a = 1/2}$$

$S_3 = (a-d)\,a\,(a+d)$

$(a-d)\,a\,(a+d) = (-1)^3 \frac{a_3}{a_0}$

$(a^2-ad)(a+d) = -6/2$

$(a^3 - ad^2) = -6/2$

$\left((1/2)^3 - (1/2)d^2\right) = -6/2$

$\frac{1}{8} - \frac{1}{2}d^2 = -3$

$-\frac{1}{2}d^2 = -3 - \frac{1}{8}$

$-\frac{1}{2}d^2 = \frac{-24-1}{8}$

$\frac{1}{2}d^2 = \frac{25}{8}$

$d^2 = \frac{24}{4}$

$d = \pm 5/2$

$a = \frac{1}{2}, \; d = 5/2$

$a-d, \; a, \; a+d$

$\frac{1}{2} - \frac{5}{2}, \; \frac{1}{2}, \; \frac{1}{2} + \frac{5}{2}$

$\frac{-4}{2}, \; \frac{1}{2}, \; \frac{6}{2}$

$-2, \; \frac{1}{2}, \; 3$

The roots are in H.P

$-\frac{1}{2}, \; 2, \; \frac{1}{3}$

$a = \frac{1}{2}, \; d = -5/2$

$a-d, \; a, \; a+d$

$\frac{1}{2} + \frac{5}{2}, \; \frac{1}{2}, \; \frac{1}{2} - \frac{5}{2}$

$6, \; 1, \; -4 \Rightarrow 3, \; \frac{1}{}, \; -2$

(5) If the roots are $x^3+px^2+qx+\lambda=0$ the G.P

S.T $\lambda p^3 = q^3$

Solution

Let the roots are $\overset{\alpha}{\frac{a}{\gamma}}, \overset{\beta}{a}, \overset{\gamma}{a\gamma}$

$$S_1 = -\frac{a_1}{a_0}$$

$$\frac{a}{\gamma} + a + a\gamma = -\frac{p}{1}$$

$$\frac{a + a\gamma + a\gamma^2}{\gamma} = -p$$

$$\frac{a(1+\gamma+\gamma^2)}{\gamma} = -p$$

$$\boxed{\frac{1+\gamma+\gamma^2}{\gamma} = -p/a} \quad -(1)$$

$$S_2 = \frac{a_2}{a_0}$$

$$\left(\frac{a}{\gamma}\right)a + a(a\gamma) + a\gamma\left(\frac{a}{\gamma}\right) = \frac{q}{1}$$

$$\frac{a^2}{\gamma} + a^2\gamma + a^2 = q$$

$$\frac{a^2 + a^2\gamma^2 + a^2\gamma}{\gamma} = q$$

$$\frac{a^2(1+\gamma^2+\gamma)}{\gamma} = q$$

$$\boxed{\frac{1+\gamma+\gamma^2}{\gamma} = q/a^2} \quad \to 0 \quad -(2)$$

$$S_3 = (-1)^3 \frac{a_3}{a_0}$$

$$\left(\frac{a}{r}\right)a(ar) = \frac{-\lambda}{1}$$

$$\boxed{a^3 = -\cancel{x}\lambda}$$

① = ②

$$\frac{-p}{a} = \frac{q}{a^2}$$

$$-p = \frac{aq}{a^2}$$

$$-p = \frac{q}{a}$$

$$-pa = q$$

$$\boxed{a = \frac{-q}{p}} \quad \text{— ③}$$

$a = -q/p$ sub in ③

$$\left(\frac{-q}{p}\right)^3 = -\lambda$$

$$\frac{-q^3}{p^3} = -\lambda$$

$$-q^3 = -\lambda p^3$$

$$q \; \boxed{\lambda p^3 = q^3}$$

(6) Solve $27x^3+42x^2-28x-8=0$ gives that its roots are in G.P.

Solution:-

let the roots be $\frac{a}{r}, a, ar$ be the roots of the g.p

~~$a_0 = 27$~~

$$S_1 = \frac{a}{r} + a + ar = -\frac{a_1}{a_0}$$

$$= -\frac{42}{27}$$

$$\frac{a+ar+ar^2}{r} = -\frac{14}{9}$$

$$\frac{a(1+r+r^2)}{r} = \frac{-14}{9} \quad -(1)$$

$$\cancel{S_2} = \left(\frac{a}{r}\right) a(ar) = \cancel{\frac{a_2}{a_0}} = \cancel{\frac{28}{27}}$$

$$\cancel{\frac{a^2}{r} + a^2r + a^2}$$

$$S_2 = \left(\frac{a}{r}\right)a + a(ar) + ar\left(\frac{a}{r}\right) = \frac{a_2}{a_0} = \frac{28}{27}$$

$$\frac{a^2}{r} + a^2r + a^2 = \frac{28}{27}$$

$$\frac{a^2(1+r^2+r)}{r} = \frac{28}{27} \quad -(2)$$

$$S_3 = \frac{a}{r} \cdot a \cdot ar = (-1)^3 \frac{a_3}{a_0} = \frac{8}{27}$$

$$a^3 = \frac{8}{27}$$

$$a = \left(\frac{8}{27}\right)^{1/3}$$

$$a = \frac{(2^3)^{1/3}}{(3^3)^{1/3}}$$

$$\boxed{a = \frac{2}{3}}$$

$$(1) \Rightarrow a\left(\frac{1+r+r^2}{r}\right) = -14/9$$

$$\frac{2}{3}\left(\frac{1+r+r^2}{r}\right) = -14/9$$

$$\frac{1+r+r^2}{r} = \frac{-14}{9} \times \frac{3}{2}$$

$$\frac{1+r+r^2}{r} = -\frac{7}{3}$$

$$1+r+r^2 = -\frac{7}{3}r$$

$$1+r+r^2+\frac{7}{3}r = 0$$

$$\frac{3+3r+3r^2+7r}{3} = 0$$

9

9 1

10

$$3r^2+10r+3 = 0$$

$$3r^2+9r+r+3 = 0$$

$$3r(r+3)+1(r+3) = 0$$

$$(3r+1)(r+3) = 0$$

$3r+1=0$, $r+3=0$

$3r=-1$ $\boxed{r=-3}$

$\boxed{r=-1/3}$

$a=2/3$, $r=-1/3$

The roots are

$\frac{a}{r}, a, ar$

$\frac{2/3}{-1/3}$, $2/3$, $(2/3)(-1/3)$

$\frac{2}{3}\times\frac{-3}{1}$, $\frac{2}{3}$, $\frac{-2}{9}$

$-2, 2/3, -2/9$

$a=2/3$, $r=-3$

The roots are

$\frac{a}{r}, a, ar$

$\frac{2/3}{-3}$, $\frac{2}{3}$, $\frac{2}{3}\times-3$

$\frac{-2}{9}$, $\frac{2}{3}$, -2

(2) Equation with the given number as the quadratic equation having roots.

α, β is $(x-\alpha)(x-\beta)=0$

(1) form a cublic equation 2 of roots are 3, $1+i\sqrt{3}$.

Solution.-

Given the roots are 3, $1+i\sqrt{3}$ & the third roots is $1-i\sqrt{3}$

∴ The equation is $(x-\alpha)(x-\beta)=0$

$$(x-3)\left[x-(1+i\sqrt{2})\right]\left[x-(1-i\sqrt{2})\right]=0$$

$$(x-3)\left[x-1-i\sqrt{2}\right]\left[x-1+i\sqrt{2}\right]=0$$

$$(x-3)\left[(x-1)^2-(i\sqrt{2})^2\right]=0$$

$$(x-3)\left[x^2+1-2x+2\right]=0$$

$$(x-3)\left[x^2-2x+3\right]=0$$

$$x^3-2x^2+3x-3x^2+6x-9=0$$

$$x^3-5x^2+9x-9=0.$$

2. find the polynomial equation of least degree having $-1, 1, 2$ & 3 as roots.

Solution:-

Given roots are $-1, 1, 2$ & 3

The equation is

$$(x+1)(x-1)(x-2)(x-3)=0$$

$$(x^2-1)(x^2-5x+6)=0$$

$$x^4-5x^3+6x^2-x^2+5x-6=0$$

$$x^4-5x^3+5x^2+5x-6=0.$$

Imaginary & irrational roots:

Theorem:

In an equation with real coeff the imaginary roots occur in pairs.

Proof:

Let $f(x) = 0$ b and n^{th} degree equation let $\alpha + i\beta$ Be one of its roots where α & β are real and $\beta \neq 0$

When then this roots is imaginary

We shall show that $\alpha - i\beta$ also is a root of $f(x) = 0$

consider,

$$[x-(\alpha+i\beta)][x-(\alpha-i\beta)] = [(x-\alpha)-i\beta][(x-\alpha)+i\beta]$$
$$= (x-\alpha)^2 + \beta^2$$

its degree is 2 divided $f(x)$ by $(x-\alpha)^2+\beta^2$ then quotient is an $(n-2)^{th}$ degree polynomial and the remainder is a first degree polynomial

Let them be $g(x)$ & $rx+s$

$$f(x) = \left[(x-\alpha)^2 + \beta^2\right] g(x) + \gamma x + s$$

$$= \left[x-(\alpha+i\beta)\right]\left[x-(\alpha-i\beta)\right] g(x) + \gamma x + s \quad \text{--- (1)}$$

Since $\alpha+i\beta$ is a root of $f(x)=0$

$$f(\alpha+i\beta)=0$$

By equ (1)

$$(0)\left[\alpha+i\beta-(\alpha-i\beta)\right] g(\alpha+i\beta) + \gamma(\alpha+i\beta)+s=0$$

$$\gamma(\alpha+i\beta)+s=0.$$

Consider real parts & imaginary part

$$\gamma\alpha+s=0 \qquad \gamma\beta=0$$

But $\beta \neq 0$ so from $\gamma\beta=0$

we get $\gamma=0$ and from $\gamma\beta+s=0$

we get $s=0$

$\therefore$ equation (1) becomes

$$f(x) = \left[x-(\alpha+i\beta)\right]\left[x-(\alpha-i\beta)\right] g(x)$$

from which we see that $\alpha-i\beta$ is roots of $f(x)=0$

Theorem: 2

In an equation with rational coefficient the irrational roots occur in pairs.

proof:

let $f(x)=0$ be the equation

let $p+\sqrt{q}$ be one of its roots where p is rational and $\sqrt{q}$ is irrational

p being rational and not equal to 0

we shall show that $p-\sqrt{q}$ also is a root

consider

$$[x-(p+\sqrt{q})][x-(p-\sqrt{q})] = [(x-p)-\sqrt{q}][x-p+\sqrt{q}]$$

$$= (x-p)^2 - q$$

its degree is to divide $f(x)$ by this then the remainder is a first degree polynomial in x

Say $rx+s$

Let the quotient be $g(x)$ then

$$f(x) = [(x-p)^2-q]\, g(x) + rs + s$$

$$= [x-(p+\sqrt{q})][x-(p-\sqrt{q})\, g(x) + rx + s \quad \text{— (1)}$$

Since $p+\sqrt{q}$ is a root of $f(x)=0$,

$$f(p+q)=0.$$

Therefore $[0]\,[p+\sqrt{q}-(p-\sqrt{q})]\;g(p+\sqrt{q})+r(p+\sqrt{q})+s=0$

$$r(p+\sqrt{q})+s=0$$

equating the ~~rr~~ rational & irrational part.

$$rp+s=0,\quad r\sqrt{q}=0.$$

but $q\neq 0$ so from the 2nd $r=0$ and then from the first $s=0$

$\therefore$ equation ① becomes.

$$f(x)=[x-(p+\sqrt{q})]\,[x-(p-\sqrt{q})]\,g(x).$$

from which we see that $p-\sqrt{q}$ is a roots of $f(x)=0$