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DEPARTMENT OF STATISTICS

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SYLLABUS

UNIT – V

Factorial experiments – Definition 22, 23 and 32 factorial experiments and their analysis – Analysis of BIBD and parametric Relations – concept of confounding and its merits.

Unit -IV

4.1. Factorial Experiments

So far we have discussed the experiments in which the effects of a single set of treatments were estimated and compared. In these experiments most of the other variable factors were kept constant. In practice, the response of biological organisms to the factor of interest is expected to differ under different levels of other factors. For example, the yield of paddy varieties may differ under different rates of fertilizer application, spacing and irrigation schedules. Hence, in agricultural research we frequently wish to what happens with a range of combination of factors. When several factors are investigated simultaneously in a single experiment, such experiments are known as factorial experiments.

Advantages of Factorial Experiment

1. It increases the scope of the experiment and its inductive value and it does so mainly by giving information not only on the main factors but on their interactions.
2. The various levels of one factor constitute replications of other factors and increase the amount of information obtained on all factors.
3. When there are no interactions, the factorial design gives the maximum efficiency in the estimate of the effects.
4. When interactions exist, their nature being unknown a factorial design is necessary to avoid misleading conclusions.
5. In the factorial design the effect of a factor is estimated at several levels of other factors and the conclusions hold over a wide range of conditions.

   Basic Ideas and Notations in the 2^n Factorial Experiment

   Let us first consider the design of the of the form 2^n in which there are n factors, each at two levels. Levels may be quite qualitative alternatives like two species of a plant. In some cases one level is simply the control group, ie., the absence of the factor and the other is its presence.

   In order to develop extended notation to present the analysis of the design in a concise form, let us start, for simplicity with a 2^2 – factorial design.

4.2. 2^2 – Factorial Design

Here we have two factors each at two levels (0,1), say, so that there are 2×2=4 treatment combinations in all. Following the notations due to Yates, let the capital letters A and B indicates the names of the two factors under study and let the small letters a and b denote one of the

two levels of each of the corresponding factors and this will be called the second level. The first level of A and B is generally expressed by the absence of the corresponding letter in the treatment combinations. The four treatment combinations can be enumerated as follows:

a_0b_0 or 1 : Factors A and B, both at first level.
a_1b_0 or a : A at second level and B at first level
a_0b_1 or b : A at first level and B at second level
a_1b_1 or ab : A and B both at second level

These four treatment combinations can be compared by laying out the experiment (i) R. B.D ., with r replicates (say), each replicate containing 4 units or (ii) 4 × 4 L. S.D., and ANOVA can be carried out accordingly. In the above cases there are 3 d.f associated with the treatment effects. In factorial experiment our main objective is to carry out separate tests for the main effects A, B and the interaction AB, splitting the treatment S.S with 3 d.f into three orthogonal components each with 1 d.f and each associated either with the main effects A and B or interactions AB.

4.2.1Main and interaction effects of 2^2 factorial design:

Suppose the factorial experiment $\mathbf{2^2}$=4 Treatment is conducted 'r' Blocks (or)Replicator.

Let [1]: total yield of the r units receiving the treatment 1.

Let [a]: Total yield of the r units receiving the treatment a.

Let [b]: Total yield of the r units receiving the treatment b.

Let [ab]: Total yield of the r units receiving the treatment ab.

(1): $\frac{[1]}{r}$ = the mean yields of the r units receiving the treatment 1.

(a): $\frac{[a]}{r}$ = the mean yields of the r units receiving the treatment a.

(b): $\frac{[b]}{r}$ = the mean yields of the r units receiving the treatment b.

(ab): $\frac{[ab]}{r}$ = the mean yields of the r units receiving the treatment ab.

The effect of A at the first level b_0 of B = $(a_1b_0)-(a_0b_0)$

= (a)-(1) --------> 1

The effect of A at the first level b_1 of B = $(a_1b_1)-(a_0b_1))$

= (ab)-(b) -------> 2

The effect of B at the first level a_0 of A = $(a_0b_1)-(a_0b_0)$

= (b)-(1) ------> 3

The effect of B at the first level a_1 of A = $(a_1b_1)-(a_1b_0)$

= (ab)-(a) ----- > 4

The main effect due to A is defined by

(2)+ (1) => A= $\frac{1}{2}$ [(ab)-(b) + (a)-(1)]

= $\frac{1}{2}$ [b (a-1) +1(a-1)]

= $\frac{1}{2}$ [(a-1) + (b+1)]

The main effect due to B is defined by

(4)+ (3) => B= $\frac{1}{2}$ [(b)-(1) + (ab)-(a)]

= $\frac{1}{2}$ [(b-1) +a (b-1)]

= $\frac{1}{2}$ [(a+1) (b-1)]

Interaction effect due to AB is defined by

(2)- (1) => AB= $\frac{1}{2}$ [(ab)-(b)-{(a)-(1)}]

= $\frac{1}{2}$ [b (a-1)-1(a-1)]

$$= \frac{1}{2}[(a-1)\ (b-1)] \text{ ---------- > } 5$$

Interaction effect due to BA is defined by

(4)- (3) => $$BA= \frac{1}{2}[(ab)-(a)-\{(b)-(1)\}]$$

$$= \frac{1}{2}\ [a\ (b-1)-1(b-1)]$$

$$= \frac{1}{2}\ [(a-1)\ (b-1)] \qquad \text{---------- > } 6$$

The equation (5) and (6) are same hence the interaction effect AB as same as BA.

4.2.3. Statistical Analysis for 2^2 Factorial experiment

Factorial Experiment are conducted either CRD, RBD, and LSD thus they can be Analysis in the usual manner except that is case treatment sum of sequence split into three orthogonal components each i.d.f. It has been already be pointed out the main effects A and B and the interaction AB. The sum of squares due to the Factorial effects A, B, and AB is obtained by multiplying by the squares of the practice these effects are usually computed from the treatment total [a], [b] and [ab] etc. The Factorial effects totals are given by the expression.

[A] = [ab]-[b] + [a]-[1]

[B] = [ab]-[a] + [b]-[1]

[AB] = [ab]-[b]- [a] + [1]

The sum of squares due to any Factorial effect is obtained by multiplying the square of the effect total by the Factor $\frac{1}{4r}$. Where r is the common replicate number sum of square due to main effect of A= $\frac{[A]^2}{4r} = S_A^2$

Similarly S.S due to main effects of B = $\frac{[B]^2}{4r} = S_B^2$

S.S due to interaction effect of AB= $\frac{[AB]^2}{4r} = S_{AB}^2$

Each with 1.D.F

Correction factor (C.F) = $\frac{G^2}{N}$

Total sum of square = $\sum_{ij \in s} y_{ij}^{\ 2} - C.F = S_T^2$

Block S.S = $\frac{\sum B_j^2}{4} - C.F = S_R^2$

S.S Treatment = $\frac{\sum T_i^2}{r} - C.F = S_t^2$

S.S Treatment = Total S.S – [S.SB+ S.ST] = S_E^2

Table 4.1: ANOVA Table

Source of variation	**Sum of square**	**Degrees of freedom**	**Mean sum of square**	**F- ratio**
Block	S_R^2	r-1	$\frac{S_R^2}{r-1} = S_R^2$	$\frac{S_R^2}{S_E^2} = F_R$
Treatment	S_t^2	3	$\frac{S_t^2}{3} = S_t^2$	$\frac{S_t^2}{S_E^2} = F_t$
Main Factor A	S_A^2	1	$\frac{S_A^2}{1} = S_A^2$	$\frac{S_A^2}{S_E^2} = F_A$
Main Factor B	S_B^2	1	$\frac{S_B^2}{1} = S_B^2$	$\frac{S_B^2}{S_E^2} = F_B$

Interaction effect AB	S_{AB}^2	1	$\frac{S_{AB}^2}{1} = S_{AB}^2$	$\frac{S_{AB}^2}{S_E^2} = F_{AB}$
Error	S_E^2	3(r-1) $(2^2$-1)(r-1)	$\frac{S_E^2}{3(r-1)} = S_E^2$	
Total	S_T^2	4r-1 $(2^2$ r-1)		

Table value:

F table [(r-1), 3(r-1)] d.f at 5% level = F_R^*

F table [3, 3(r-1)] d.f at 5% level = F_t^*

F table [1, 3(r-1)] d.f at 5% level = F^*

Conclusion:

(i) $F_R < F_R^*$ we need not reject and we conclude that there is no significance difference between replicates.

(ii) $F_t < F_t^*$ we need not reject and we conclude that there is no significance difference between Treatment.

(iii) $F_A < F_A^*$ we need not reject and we conclude that there is no significance

(iv) $F_B < F_B^*$ we need not reject and we conclude that there is no significance difference main effect B.

(v) $F_{AB} < F^*$ we need not reject and we conclude that there is no significance difference interaction effect AB.

Table 4.2: Yates Method for 2×2 Factorial experiment:

Treatment combination	**Total yield from all replicates (2)**	**(3)**	**(4)**	**Effect total**
1	[1]	[1]+[a]	[1]+[a]+[b]+[ab]	Ground total
A	[a]	[b]+[ab]	[a]-[1]+[ab]-[b]	[A]
B	[b]	[a]-[1]	[b]+[ab]-[1]-[a]	[B]
Ab	[ab]	[ab]-[b]	[ab]-[b]+[a]-[1]	[AB]

Problem 1:

Find out ,the effect and interaction effect in the following 2 Factorial experiment and write dawn the analysis of variance table;-

Table 4.3 ANOVA Table

Block	Treatment	(a)	(b)	(ab)
	00	10	01	11
I	64	25	30	10
II	25	14	50	33

III	76	12	41	17
IV	75	33	25	10

Solution:

Null hypothesis H_{ob}**:**

There is no significance difference between blocks.

H_{ot}: There is no significance difference between Treatments.

H_{oA}: There is no significance difference between main effect A.

H_{oB}: There is no significance difference between main effect B.

H_{oAB}: There is no significance difference between effect AB.

Treatment Combination	Total yield from all blocks	(3)	Effects Total	S.S
1	240	324	540	$\frac{(540)^2}{16} = 18225$
a	84	216	-232	$\frac{(-232)^2}{16} = 3364$
b	146	-156	-108	$\frac{(-108)^2}{16} = 729$
ab	70	-76	80	$\frac{(80)^2}{16} = 400$

C.F = $\frac{G^2}{N} = \frac{(540)^2}{16} = 18225$

C.F = 18225

Total sum of square = $\sum_{ij\in s} y_{ij} - C..F = (64)^2 + (25)^2 + ...(25)^2 + (10)^2 - C..F$

= 25460- 18225

= 7235

Sum of square due to Block = $\frac{\sum B_j^2}{4} - C.F$

= $\frac{73290}{4} - 18225$

S.SB = 97.5

Sum of square due to Treatment = $\frac{\sum T_i^2}{4} - C.F = \frac{90872}{4} - 18225$

S.S.T_r = 4493

Sum of square due to error = T.S.S – (S.S.B + S.S.T_r)

= 7235- (97.5+4493)

S.S.E = 2633.5

Table 4.3: Yates method for 2^2 Factorial experiment:

Source of variation	Sum of square	Degree of freedom	Mean sum of square	F- ratio
Block	97.5	3	32.5	0.1106 F_R
Treatments	4493	3	1497.67	5.0975 F_T
Main effect A	3364	1	3364	11.4499 F_A

Main effect B	729	1	729	2.48 F_B
Interaction effect of AB	400	1	400	1.3614 F_{AB}
Error	2644.5	9	293.8	-
Total	7235	15	-	-

F- Table value:

F (3.9) d.f at 5% level = 3.86 → F_A^*

F (1.9) d.f at 5% level = 5.12→ F_B^*

Conclusion:

(i) $F_R < F_A^*$ we need not reject H_0 and we conclude that there is no significance difference between blocks.

(ii) $F_T > F_A^*$ we need not accept H_0 and we conclude that there is no significance difference between Treatments.

(iii) $F_A > F_B^*$ We need not accept H_0 and we conclude that there is no significance difference between main effect A.

(iv) $F_B < F_B^*$ We need not reject H_0 and we conclude that there is no significance difference between main effect B.

(v) $F_{AB} < F_B^*$ We need not reject H_0 and we conclude that there is no significance difference between interaction effect AB.

4.3. 2^3 Factorial Experiment

(main and interaction effect of 2^3 factorial experiment) suppose that are factorial experiment $2^3=8$ Treatment combinations is conducted 'r' Blocks (or) replicates.

[1] Total of r units receiving the treatment 1.

[a] Total of r units receiving the treatment a.

[b] Total of r units receiving the treatment b.

[ab] Total of r units receiving the treatment ab.

[c] Total of r units receiving the treatment c.

[ac] Total of r units receiving the treatment ac.

[bc] Total of r units receiving the treatment bc.

[abc] Total of r units receiving the treatment abc.

Simple effect of A:-

Level of B	Level of C	Effect of A
b_0	c_0	$a_1b_0c_0 - a_0b_0c_0$ = (a)-(1)
b_1	c_0	$a_1b_1c_0 - a_0b_1c_0$ = (ab)-(b)

b_0	c_1	$a_1b_0c_1$ - $a_0b_0c_1$ = (ac)-(c)
b_1	c_1	$a_1b_1c_1$ - $a_0b_1c_1$ = (abc)-(bc)

Simple effect of A

$$= \frac{1}{4}[(abc)-(bc)+(ac)-(c)+(ab)-(b)+(a)-(1)]$$

$$= \frac{1}{4}[bc(a-1)+c(a-1)+b(a-1)+1(a-1)]$$

$$= \frac{1}{4}[(a-1)(bc+c+b+1)]$$

$$= \frac{1}{4}[(a-1)\{c(b+1)+1(b+1)\}]$$

$$= \frac{1}{4}[(a-1)\{(b+1)(c+1)\}]$$

$$= \frac{1}{4}[(a-1)(b+1)(c+1)]$$

S.E. of A = $\frac{1}{4}[(a-1)(b+1)(c+1)]$

Simple effect of B:

Level of A	Level of C	Effect of B
a_0	c_0	$a_0b_1c_0$ - $a_0b_0c_0$ = (b)-(1)
a_1	c_0	$a_1b_1c_0$ - $a_1b_0c_0$ = (ab)-(a)
a_0	c_1	$a_0b_1c_1$ - $a_0b_0c_1$ = (ac)-(a)

a_1

c_1 $a_1b_1c_1$ - $a_1b_0c_1$ = (abc)-(ac)

Simple effect of B

$$= \frac{1}{4}[(abc)-(ac)+(bc)-(c)+(ab)-(a)+(b)-(1)]$$

$$= \frac{1}{4}[ac(b-1)+c(b-1)+a(b-1)+1(b-1)]$$

$$= \frac{1}{4}[(b-1)(ac+c+a+1)]$$

$$= \frac{1}{4}[(b-1)\{c(a+1)+1(a+1)\}]$$

$$= \frac{1}{4}[(b-1)\{(a+1)(c+1)\}]$$

$$= \frac{1}{4}[(b-1)(a+1)(c+1)]$$

S.E. of B $= \frac{1}{4}[(b-1)(a+1)(c+1)]$

Simple effect of C:-

Level of A	Level of B	Effect of C

a_0	b_0	$a_0b_0c_1$ - $a_0b_0c_0$ = (c)-(1)
a_1	b_0	$a_1b_1c_0$ - $a_1b_0c_0$ = (ac)-(a)
a_0	b_1	$a_0b_1c_1$ - $a_0b_1c_0$ = (bc)-(b)
a_1	b_1	$a_1b_1c_1$ - $a_1b_1c_0$ = (abc)-(ab)

Simple effect of C

$$= \frac{1}{4}[(abc)-(ab)+(bc)-(b)+(ac)-(a)+(c)-(1)]$$

$$= \frac{1}{4}[ab(c-1)+b(c-1)+a(c-1)+1(c-1)]$$

$$= \frac{1}{4}[(c-1)(ab+b+a+1)]$$

$$= \frac{1}{4}[(c-1)\{b(a+1)+1(a+1)\}]$$

$$= \frac{1}{4}[(c-1)\{(a+1)(b+1)\}]$$

$$= \frac{1}{4}[(c-1)(a+1)(b+1)]$$

S.E. of C = $\frac{1}{4}[(a+1)(b+1)(c-1)]$

Interaction effect of AB:

AB = $\frac{1}{4}[\{(abc)-(bc)+(ab)-(b)\}-\{(ac)-(c)+(a)-(1)\}]$

$$= \frac{1}{4}[(abc)-(bc)+(ab)-(b)-(ac)+(c)-(a)+(1)]$$

(In main effect A we are add b_1 terms and substact b_0 terms)

$$= \frac{1}{4}[bc(a-1)+b(a-1)-c(a-1)-1(a-1)]$$

$$= \frac{1}{4}[(a-1)(bc+b-c-1)]$$

$$= \frac{1}{4}[(a-1)\{b(c+1)-1(c+1)\}]$$

$$= \frac{1}{4}[(a-1)(b-1)(c+1)]$$

I.E of B $= \frac{1}{4}[(a-1)(b-1)(c+1)]$

Interaction effect of BC:

AC = $\frac{1}{4}[\{(abc)-(bc)+(ab)-(b)\}-\{(ac)-(c)+(a)-(1)\}]$

(In main effect A we are add c_1 terms and subtract c_0 terms)

$$= \frac{1}{4}[(abc)-(bc)+(ac)-(c)-(ab)+(b)-(a)+(1)]$$

$$= \frac{1}{4}[bc(a-1)+c(a-1)-b(a-1)-1(a-1)]$$

$$= \frac{1}{4}[(a-1)(bc+c-b-1)]$$

$$= \frac{1}{4}[(a-1)\{c(b+1)-1(b+1)\}]$$

$$= \frac{1}{4}[(a-1)(b+1)(c-1)]$$

I.E. of AC = $\frac{1}{4}[(a-1)(b+1)(c-1)]$

Interaction effect of BC

BC = $\frac{1}{4}[\{(abc)-(ac)+(bc)-(c)\}-\{(ab)-(a)+(b)-(1)\}]$

(In main effect B we are add c_1 terms and subtract c_0 terms)

$$= \frac{1}{4}\left[(abc)-(ac)+(bc)-(c)-(ab)+(a)-(b)+(1)\right]$$
$$= \frac{1}{4}\left[ac(-1)+c(b-1)-a(b-1)-1(b-1)\right]$$
$$= \frac{1}{4}\left[(b-1)(ac+c-a-1)\right]$$
$$= \frac{1}{4}\left[(b-1)\{c(a+1)-1(a+1)\}\right]$$
$$= \frac{1}{4}\left[(b-1)(a+1)(c-1)\right]$$

Interaction effect of AC = $\frac{1}{4}\left[(a+1)(b-1)(c-1)\right]$

Interaction effect of ABC:

Interaction effect of AB at C_0 of C = $\frac{1}{2}\left[(a_1b_1-a_0b_1)c_0-(a_1b_0-a_0b_0)c_0\right]$

Interaction effect of AB at C_1 0f C = $\frac{1}{2}\left[(a_1b_1-a_0b_1)c1-(a_1b_0-a_0b_0)c_1\right]$

Interaction effect of ABC = $\frac{1}{4}\left[(abc)-(bc)-(ac)+((c)-(ab)+(b)+(a)-(1))\right]$

$$= \frac{1}{4}\left[bc(a-1)-c(a-1)-b(a-1)+1(a-1)\right]$$
$$= \frac{1}{4}\left[(a-1)(bc-c-b+1)\right]$$
$$= \frac{1}{4}\left[(a-1)\{c(b-1)-1(a-1)\}\right]$$
$$= \frac{1}{4}\left[(a-1)(b-1)(c-1)\right]$$

I.E of ABC = $\frac{1}{4}\left[(a1)(b-1)(c-1)\right]$

4.3.1. Analysis of 2^3 Factorial Design:

$$Y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\beta\alpha)_{jk} + (\alpha\gamma)_{ik} + (\alpha\beta\gamma)_{ijk} + \rho_l + \varepsilon_{ijkl}$$

Where, Y_{ijkl} is the yield from the ith row j^{th}Block μ is the general mean effect.

α_i is the effect due to i^{th} level of Treatment A.

β_j is the effect due to j^{th} level of Treatment B.

γ_k is the effect due to k^{th} level of Treatment C.

$(\alpha\beta)_{ij}$ is the interaction effect due to i and j^{th} level of Treatment AB.

$(\beta\gamma)_{ij}$ is the interaction effect due to j and k^{th}level of Treatment BC.

$(\alpha\gamma)_{ik}$ is the interaction effect due to i and k^{th} level of Treatment AC.

$(\alpha\beta\gamma)_{ijk}$ is the interaction effect due to i,j and k^{th} level of Treatment ABC.

$\rho_l$is the effect of due to the lth Block.

$\varepsilon_{ijkl}$is the error effect.

The above parameter are subject to following restrictions

$\Sigma\alpha_i=0$, $\Sigma(\alpha\beta)_{ij}=0$

$\Sigma\beta_j=0$, $\Sigma(\beta\alpha)_{ij}=0$

$\Sigma(\alpha\beta\gamma)_{ijk}=0$, $\Sigma(a\gamma)_{ik}=0$

$\Sigma\rho_l=0$

Null Hypothesis:

H_{OR}: - there is no significance difference between replicates.

H_{OT}: - there is no significance difference between Treatment (or) Factorial not present.

Statistical Analysis for 2^3 factorial experiment

The sum of square due to any factorial effect is obtain as multiplying the square of the effect total by the factor $\frac{1}{8r}$, where r is common replicate number.

Sum of square due to main effect A = $\frac{[A]^2}{8r}$

Sum of square due to main effect B = $\frac{[B]^2}{8r}$

Sum of square due to interaction effect AB = $\frac{[AB]^2}{8r}$

Sum of square due to interaction effect A = $\frac{[AC]^2}{8r}$

Sum of square due to interaction effect C = $\frac{[C]^2}{8r}$

Sum of square due to interaction effect BC = $\frac{[BC]^2}{8r}$

Sum of square due to interaction effect ABC = $\frac{[ABC]^2}{8r}$ each with 1 d.f

Table 4.6: Yates method for 2^3 factorial experiment: -

Treatment Combination	Treatment Total	(3)	(4)	(5)	Effect total (6)
1	[1]	[1]+[a]	[1]+[a]+[b]+[ab]	[1]+[a]+[b]+[ab]+[c]+[ac]+[bc+[abc]	G
a	[a]	[b]+[ab]	[c]+[ac]+[bc]+[abc]	[a]-[1]+[ab]-[b]+[ac]+[ac-[c]+[abc]-[bc]	[A]
b	[b]	[c]+[ac]	[a]-[1]+[ab]-[b]	[b]+[ab]-[1]-[a]+[bc]+[abc]-[c]-[ac]	[B]
ab	[ab]	[bc]+[abc]	[ac]-[c]+[abc]-[bc]	[ab]-[b]-[a]+[1]+[abc]-[bc]-[ac]+[c]	[AB]
c	[c]	[a]-[1]	[b]+[ab]-[1]-[a]	[c]+[ac]+[bc]+[abc]-[1]-[a]-[b]-[ab]	[C]
ac	[ac]	[ab]-[b]	[bc]+[abc]-[c]-[ac]	[ac]-[c]+[abc]-[bc]-[a]+[1]-[ab]+[b]	[AC]

bc	[bc]	[ac]-[c]	[ab]-[b]-[a]+[1]	[bc]+[abc]-[c]-[ac]-[b]-[ab]+[1]+[a]	[BC]
abc	[abc]	[abc]-[bc]	[abc]-[bc]-[ac]+[c]	[abc]-[bc]-[ac]+[c]-[ab]+[b]+[a]-[1]	[ABC]

Sum of variance	**Sum of square**	**d.f**	**Mean sum of square**	**F- ratio**
Replication	S_R^2	r-1	$\frac{S_R^2}{r-1} = S_R^2$	$\frac{S_R^2}{S_E^2} = F_R$
Treatment	S_t^2	7	$\frac{S_t^2}{7} = S_t^2$	$\frac{S_t^2}{S_E^2} = F_t$
Main effect A	S_A^2	1	$\frac{S_A^2}{1} = S_A^2$	$\frac{S_A^2}{S_E^2} = F_A$
Main effect B	S_B^2	1	$\frac{S_B^2}{1} = S_B^2$	$\frac{S_B^2}{S_E^2} = F_B$
Int. effect AB	S_{AB}^2	1	$\frac{S_{AB}^2}{1} = S_{AB}^2$	$\frac{S_{AB}^2}{S_E^2} = F_{AB}$
main effect C	S_C^2	1	$\frac{S_C^2}{1} = S_C^2$	$\frac{S_C^2}{S_E^2} = F_C$
Int. Effect AC	S_{AC}^2	1	$\frac{S_{AC}^2}{1} = S_{AC}^2$	$\frac{S_{AC}^2}{S_E^2} = F_{AC}$
Int. Effect BC	S_{BC}^2	1	$\frac{S_{BC}^2}{r-1} = S_{BC}^2$	$\frac{S_{BC}^2}{S_E^2} = R_{BC}$

Int. Effect ABC	S_{ABC}^2	1	$\frac{S_{ABC}^2}{r-1} = S_{ABC}^2$	$\frac{S_{ABC}^2}{S_E^2} = R_{ABC}$
Error	S_E^2	7r-1)	$\frac{S_E^2}{7(r-1)} = S_E^2$	-
Total	S_T^2	8r-1	-	-

Table value:

F [(r-1), (7(r-1))] at 5% level = F_1

F [7, 7(r-1)] at 5% level = F_2

F [1, 7(r-1)] at 5% level = F_3

Conclusion:

(i) $F_R < F_1$ we need not reject the null hypothesis H_0 and we conclude that there is no significance difference between replicates.

(ii) $F_t < F_2$ we need not reject the null hypothesis H_0 and we conclude that there is no significance difference between Treatments.

(iii) $F_A < F_3$ we need not reject the null hypothesis H_0 and we conclude that there is no Factorial effect in main effect A.

(iv) $F_B < F_3$ we need not reject the null hypothesis H_0 and we conclude that there is no Factorial effect in main effect B

(v) $F_{AB} < F_3$ we need not reject the null hypothesis H_0 and we conclude that there is no Factorial effect in interaction effect AB.

(vi) $F_C < F_3$ we need not reject the null hypothesis H_0 and we conclude that there is no Factorial effect in main effect C.

(vii) $F_{AC} < F_3$ we need not reject the null hypothesis H_0 and we conclude that there is no Factorial effect in interaction effect AC.

(viii) $F_{BC} < F_3$ we need not reject the null hypothesis H_0 and we conclude that there is no Factorial effect in interaction BC.

(ix)$F_{ABC} < F_3$ we need not reject the null hypothesis H_0 and we conclude that there is no Factorial effect in interaction effect ABC.

4.4. 2^n – Factorial Experiment

The results and the notations of 2^2 and 2^3 can be generalized to the case of 2^n experiment. g Here we consider n factors each at 2 levels. Suppose A, B, C, D,…, K are the factors each at two levels(0, 1). Corresponding small letters a, b, c, d,…,k denote the corresponding factors at the second level, the first level of any factor being signified by the absence of the corresponding small letter. The treatment combinations, in standard order can be written as:

1, a, b, ab, c, ac, bc, abc, d, ad, bd, abd, cd, acd, bcd, abcd, etc.

For 2^n-experiment, the various factorial effects are enumerated as follows:

Main effects : n_{C_1} in number

Two-factor interactions : n_{C_2} in number

Three –factor Interactions : n_{C_3} in number

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.

.

n factor interaction : n_{C_n} in number

Hence, the total number of factorial effects in 2n –experiment are

n-factor interactions ABCD…K	$S^2_{AB\cdots K}$ = $[AB\ldots K]^2/r.2^n$	1	$s^2_{AB\ldots K} = S^2_{AB\ldots K}$
Error	S^2_E = By subtraction	$(r-1)(2^n-1)$	$s^2_E = \frac{S^2_E}{(r-1)(2^n-1)}$
Total	Raw.S.S- C.F	$r.2^{n-1}$	

The block effects and the factorial effects (main and interactions) can be tested for significance by comparing their means S.S with error S.S.

UNIT- V

5.1 Confounding

When only the portions of treatment combinations are allotted to block within a replication, the comparison between blocks in a replication represents some treatment comparison, either a main effect or on interaction. In such cases it is not possible to distinguish treatment comparisons from block comparisons. Such a mix up is termed as confounding.

Advantages and disadvantages of Confounding

The only and the greatest advantage of confounding scheme lies in the fact that it reduces the experimental error considerably by stratifying the experimental material into homogeneous sub sets or sub groups. The removal of the variation among incomplete blocks within replicates often results in smaller error mean square as compared with a randomised complete block design, thus making the comparisons among some treatments more precise.

The following are the disadvantages of confounding

1. The confounded contrasts are replicated fewer times than are the other contrasts and as such there is loss of information on them and they can be estimated with a lower degree of precisions as the number of replications for them is reduced.

2. The algebraic calculations are usually more difficult and the statistical analysis is complex, specifically when some of the units are missing.
3. A number of problems arise if the treatments interact with blocks.

5.2 Partial and complete confounding

The same interactions are confounding in each replication (or) different sets of interaction are confounded in different replications. Both the procedures are practised if the same set of the interactions is confounded in all the replications confounding are called complete confounding.

If again different sets of interaction are confounded in different replications confounding is called partial confounding

5.3 Complete confounding 2^3 experiments

In a 2^3 experiment the 8 treatment combination require 8 units of homogeneous material each two from a block.

For example:Let us consider confounding the highest order interactions ABC we know that the interaction effect ABC is given by

$$\text{ABC} = \frac{1}{4}\left[(abc)-(bc)-(ac)+(c)-(ab)+(b)+(a)-(1)\right]$$

$$= \frac{1}{4}\left[(abc)+(a)+(b)+(c)-(ab)-(bc)-(ac)-(1)\right]$$

ABC confounded with blocks.

Block 1: (1) (ab) (ac) (bc)

Replicate

Block 2: (a) (b) (c) (abc)

Yates method for a 2^3 experiment:

Black size is four in the question

Block 1 1 (ab)

Block 2 (ac) (bc)

Block 3 (a) (b)

Block 4 (c) (abc)

Table 5.1

Treatment combination	**Treatment Total**	**(3)**	**(4)**	**(5)**	**Effects total**
1	[1]	$[1]+[a]=u_1$	$u_1+u_2=v_1$	$v_1+v_2=w_1$	G.T
a	[a]	$[b]+[ab]=u_2$	$u_3+u_4=v_2$	$v_3+v_4=w_2$	[A]
b	[b]	$[c]+[ac]=u_3$	$u_5+u_6=v_3$	$v_5+v_6=w_3$	[B]
ab	[ab]	$[bc]+[abc]=u_4$	$u_7+u_8=v_4$	$v_7+v_8=w_4$	[AB]
c	[c]	$[a]-[1]=u_5$	$u_2-u_1=v_5$	$v_2-v_1=w_5$	[C]
ac	[ac]	$[ab]-[b]=u_6$	$u_4-u_3=v_6$	$v_4-v_3=w_6$	[AC]
bc	[bc]	$[ac]-[c]=u_7$	$u_6-u_5=v_7$	$v_6-v_5=w_7$	[BC]
abc	[abc]	$[abc]-[bc]=u_8$	$u_8-u_7=v_8$	$v_8-v_7=w_8$	Not estimatle

This confound component contain in the (2r-1) d.f. The ANOVA table will be as follows.

Table 5.2 : ANOVA Table:

S.V	S.S	d.f	M.S.S	F- ratio
Blocks	S_b^2	2r-1	S_b^2	$\frac{S_b^2}{S_E^2}=F_1$
Treatment	S_t^2	6	S_t^2	$\frac{S_t^2}{S_E^2}=F_2$

A	S_A^2	1	S_A^2	$\frac{S_A^2}{S_E^2} = F_3$
B	S_B^2	1	S_B^2	$\frac{S_B^2}{S_E^2} = F_4$
AB	S_{AB}^2	1	S_{AB}^2	$\frac{S_{AB}^2}{S_E^2} = F_5$
C	S_C^2	1	S_C^2	$\frac{S_C^2}{S_E^2} = F_6$
AC	S_{AC}^2	1	S_{AC}^2	$\frac{S_{AC}^2}{S_E^2} = F_7$
BC	S_{BC}^2	1	S_{BC}^2	$\frac{S_{BC}^2}{S_E^2} = F_8$
Error	S_E^2	6(r-1)	S_E^2	-
Total	S_T^2	8r-1	-	-

Null Hypothesis: -

H_0: confounding is not effective

H_1: confounding is effective

Inference:

If the calculated F value is less than the tabulated F^* value we accept H_0. Otherwise we reject H_0.

5.4 Analysis of 2^3 partial confounding

The analysis of 2^3 partially confounded design differs from that of the ordinary 2^3 factorial experiment replicated 4 times only in the calculation of the partially confounded interactions. Each interaction being estimated only from the three replicates in which the given interaction is not confounded.

Analysis of 2^3 partially confounded design with four replications and 'r' such replications. Let us suppose that a number of repetitions say 'r', of the above pattern or layout are performed such the positions of the replication, Blocks within replications and Blocks within Blocks are randomised then the structure of the Anova table will be as follows.

Table 5.3

S.V	d.f	S.S
Blocks	8r-1	$\frac{1}{4}\Sigma(\text{total of Blocks})^2 - \frac{G^2}{32r}$
Treatment	7	$S_A^2 + S_B^2 + S_C^2 + S_{AB}^2 + S_{BC}^2 + S_{AC}^2 + S_{ABC}^2 = S_t^2$
A	1	$[A]^2/32$ r
B	1	$[B]^2/32$ r
C	1	$[C]^2/32$ r
AB	1	$[AB]^2/24$ r

AC	1	$[AC]^2/24\ r$
BC	1	$[BC]^2/24\ r$
ABC	1	$[ABC]^2/24\ r$
Error	24r-7	(By difference)= S_E^2
Total	32r-1	S_T^2 = Total S.S

Calculation of sum of square due to confounded effects:

It has already been explained that sum of square for confounded effects are to be obtained from those replications only in which the given effect is not confounded from practical point of view these sum of square can be obtain from the table of Yates method for all the 4 replications by appling sum adjusting factor (A.F) for any confounded effects is computed as follows

(i) Note the replication in which the given effect is confounded.

(ii) Note the sign of (1) in the corresponding algebraic expression of the effect to be confounded.

If the sign is positive then

A.F = [Total of the block containing (1) of replicate in which the Effect is confounded] – [Total of the block not containing (1) of the replicate in which the effect is confounded]

A.F = $T_1 - T_2$ (say)

If the sign is negative,

A.F = $T_2 - T_1$

This adjusting factor will be subtracted from the Factorial effects total of the confounded effects obtained from Yates's method for all replicates.

5.5 Balanced Incomplete Block Designs(BIBD)

If in a block the number of experimental units or plots is smaller than the number of treatments, then the block is said to be incomplete and a design constituted of such blocks is called an incomplete block design.

Balanced incomplete block designs which were developed for experiments in plant breeding and agriculture selection comparisons among pairs of treatments is made with equal precision.

Definitions

Incomplete Block Design(I.B.D)

An incomplete block design is one having v treatments and b blocks each of size k such that each of the treatments is replicated r times and each pair of treatments occurs once and only once in the same blocks v, b, r, and k are known as the parameters of the I.B.D

Balanced Incomplete Block Design

An arrangement of v treatments in b blocks of k plots each($k<v$) is known as BIBD, if

i) Each treatment occurs once and only once in r blocks and
ii) Each pair of treatments occurs together in λ blocks.
BIBD is used when all treatment comparisons are equally important as it ensures equal precisions of the estimates of all pairs of treatment effects.

Parameters of BIBD

The integers v,r,b,k and λ are called the parameters of the BIBD., where

V = number of varies or treatments, b=number of blocks

K=block size , r= number of replicates for each treatment

λ= number of blocks in which any pair of treatments occurs together or number of times any two treatments occur together in a block. The following parametric relations serve as a necessary condition for the existence of a BIBD.

i) $Vr = bk$ ii) $\lambda(v-1) = r(k-1)$ iii) $b \geq v$(Fisher's Inequality)

Theorem 5.1

$Vr = bk$

Proof:

Since there are v treatments each replicated r times, total number of plots in the design is vr. Further since there are b blocks each of size k, there are bk plots in all.

Hence vr=bk

Incidence Matrix: Associated with any design D is the incidence matrix N = (n_{ij}), (i = 1,2,…,v; j = 1,2,…,b), where n_{ij} denotes the number of times the ith treatment occurs in the jth block. Thus by the definition of a BIBD

$$N = \begin{bmatrix} n_{11} & n_{12} & \dots & n_{1b} \\ n_{21} & n_{22} & \dots & n_{2b} \\ \vdots & \vdots & \vdots & \vdots \\ n_{v1} & n_{v2} & \dots & n_{vb} \end{bmatrix} \qquad \text{…(5.1)}$$

Where n_{ij} =1, if i^{th} treatment occurs in the jth block …(5.2)

=0, otherwise

We also observe, by definition of BIBD

$$\sum_{j=1}^{b} n_{ij} = \sum_{j=1}^{b} n_{ij}^2 = r; (i = 1, 2, \dots, v) \qquad \text{… (5.3)}$$

$$\sum_{i=1}^{v} n_{ij} = \sum_{i=1}^{v} n_{ij}^2 = k; (j = 1, 2, \dots, v) \qquad \text{… (5.4)}$$

$$\sum_{j=1}^{b} n_{ij} n_{ij} = \lambda; (i \neq l = 1, 2, .., v) \qquad \text{…(5.5)}$$

Since $n_{ij}n_{lj}$=1 if and only if i^{th} and l^{th} treatments occur together in the jth block otherwise it is zero and they occur together in λ blocks

If N' denotes the transpose of N then

$$NN' = \begin{bmatrix} \sum_j n_{1j}^2 & \sum_j n_{1j}n_{2j} & \cdots & \sum_j n_{1j}n_{vj} \\ \sum_j n_{2j}n_{1j} & \sum_j n_{2j}^2 & \cdots & \sum_j n_{2j}n_{vj} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_j n_{vj}n_{1j} & \sum_j n_{vj}n_{2j} & \cdots & \sum_j n_{vj}^2 \end{bmatrix}$$

$$= \begin{bmatrix} r & \lambda & \lambda & \cdots & \lambda \\ \lambda & r & \lambda & \cdots & \lambda \\ \vdots & \vdots & \vdots & & \vdots \\ \lambda & \lambda & \lambda & \cdots & r \end{bmatrix}_{v\times v} \qquad \text{.... (5.6)}$$

Theorem 5.2λ(v-1)=r(k-1) ...(5.7)

Proof

Let us denote by E_{mn} the m×n matrix all of whose elements are unity. From (5.6), we get

$$NN'E_{v1} = \begin{bmatrix} r & \lambda & \lambda & \cdots & \lambda \\ \lambda & r & \lambda & \cdots & \lambda \\ \vdots & \vdots & \vdots & & \vdots \\ \lambda & \lambda & \lambda & \cdots & r \end{bmatrix}_{v\times v} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{v\times 1}$$

$$= [r + \lambda(v-1)] \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Also =[r+λ(v-1)]E_{v1} ...(5.8)

$$NN'E_{v1} = N(N'E_{v1})$$

From (5.4)

$$= N \begin{bmatrix} n_{11} & n_{21} & \cdots & n_{v1} \\ n_{12} & n_{22} & \cdots & n_{v2} \\ \vdots & \vdots & & \vdots \\ n_{1b} & n_{2b} & & n_{vb} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$= N\begin{bmatrix} \sum_i n_{i1} \\ \sum_i n_{i2} \\ \vdots \\ \sum_i n_{ib} \end{bmatrix} \quad = \text{N } N\begin{bmatrix} k \\ k \\ \vdots \\ k \end{bmatrix}$$

From (5.3)

$$= \text{k}\begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1b} \\ n_{21} & n_{22} & \cdots & n_{2b} \\ \vdots & \vdots & & \vdots \\ n_{v1} & n_{v2} & \cdots & n_{vb} \end{bmatrix}_{v\times b}\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{b\times 1}$$

$$= k\begin{bmatrix} \sum_j n_{1j} \\ \sum_j n_{2j} \\ \vdots \\ \sum_j n_{vj} \end{bmatrix} = \text{k } k\begin{bmatrix} r \\ r \\ \vdots \\ r \end{bmatrix}_{v\times 1} \qquad \text{...(5. 9)}$$

$=kr\ E_{v1}$ ... (5.10)

From (5.9) and (5.10), we get

$[r+\lambda(v-1)]E_{v1} = kr\ E_{v1}$

$r+\lambda(v-1) = kr$

i.e., $\lambda(v-1) = r(k-1)$

Theorem 5.3 b ≥v (Fisher's Inequality)

Proof. From (5.6) the determinant of the matrix $|NN'| = \begin{bmatrix} r & \lambda & \lambda & \dots & \lambda \\ \lambda & r & \lambda & \dots & \lambda \\ \vdots & \vdots & \vdots & & \vdots \\ \lambda & \lambda & \lambda & \cdots & r \end{bmatrix}_{v\times v}$

Adding 2nd , 3rd , …, vth columns to the first column and taking [r+(v-1)] common from the first column, we get

$$NN' = [r+(v-1)\lambda]\begin{bmatrix} 1 & \lambda & \lambda & \cdots & \lambda \\ 1 & r & \lambda & \cdots & \lambda \\ 1 & \lambda & r & \cdots & \lambda \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \lambda & \lambda & \cdots & r \end{bmatrix}$$

$$[r+(v-1)\lambda]\begin{bmatrix} 1 & \lambda & \lambda & \cdots & \lambda \\ 0 & (r-\lambda) & 0 & \cdots & 0 \\ 0 & 0 & (r-\lambda) & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & (r-\lambda) \end{bmatrix}$$

(subtracting first row from the 2nd, 3rd, …, vth row)

$=[r+(v-1)\lambda(r-\lambda)^{v-1}$ expanding by first column)

$=rk(r-\lambda)^{v-1}$ using (5.6)

Thus $|NN'| \neq 0$, for if r = λ then from (5.6) we get

$$(v-1) = (k-1) \quad \Rightarrow \quad v = k$$

Indicating that the design reduces to randomized block design. Hence, NN' is non singular and consequently $Rank(NN') = v$ …(5.11)

Since v is the order of matrix NN'.

$$Rank(NN') = Rank(N)$$

$\therefore \quad Rank(N) = v$ **from (5.11)** **… (5.12)**

But since N is a v × b matrix, it ranks can be at most b.

V= rank N ≤ b b ≥ v as desired. …(5.13)

Theorem 5.4

(i) r ≥ k
(ii) b ≥ v+r-k

proof

(i) we have vr = bk $\Rightarrow r = \frac{b}{v}k$

since b ≥ v, we get r ≥ k

(ii) we have v-k ≥ 0 and r-k ≥ 0

$$\therefore \quad (v-k)(r-k) \geq 0 \quad \Rightarrow \left(\frac{v}{k}-1\right)(r-k) \geq 0$$

i.e., $\frac{v}{k}(r-k)-(r-k) \geq 0$

$$\therefore \frac{vr}{k}-v \geq r-k \qquad \Rightarrow b \geq v+r-k \qquad [\because vr = bk]$$

Symmetric BIBD

Definition. A BIBD is said to be symmetric if b = v and r = k.

Theorem 5.5 In a symmetric BIBD, the number of treatments common between any two blocks is λ

Proof.

We have

$$NN' = \begin{bmatrix} r & \lambda & \lambda & \cdots & \lambda \\ \lambda & r & \lambda & \cdots & \lambda \\ \vdots & \vdots & \vdots & & \vdots \\ \lambda & \lambda & \lambda & & r \end{bmatrix}$$

$$= \begin{bmatrix} r-\lambda & 0 & 0 & \cdots & 0 \\ 0 & r-\lambda & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & & r-\lambda \end{bmatrix} + \begin{bmatrix} \lambda & \lambda & \cdots & \lambda \\ \lambda & \lambda & \cdots & \lambda \\ \vdots & \vdots & & \vdots \\ \lambda & \lambda & & \lambda \end{bmatrix}$$

$=(r-\lambda)I_v+\lambda E_{vv}$

Where Iv is a unit matrix of order v.

Also for a symmetric BIBD, we have

$$(NN')^{-1} = \left[I_v - \frac{\lambda}{r^2}E_{vv}\right]$$

$$\Rightarrow (N')^{-1}N^{-1} = \frac{1}{r-\lambda}\left[I_v - \frac{\lambda}{r^2}E_{vv}\right]$$

Premultiplying by (N'), we get $N^{-1} = \frac{1}{r-\lambda}\left[N' - \frac{\lambda}{r^2} N'E_{vv}\right]$ …(5.14)

But it can be easily verified that for symmetric BIBD

N'Evv=NEvv=rEvv=k Evv

$$\frac{N'}{r} E_{vv} = E_{vv}$$

Substituting in (5.14), we get $N^{-1} = \frac{1}{r-\lambda}\left[N' - \frac{\lambda}{r} E_{vv}\right]$

Post multiplying by N, we have

$$I_v = \frac{1}{r-\lambda}\left[N'N - \frac{\lambda}{r} NE_{vv}\right] = \frac{1}{r-\lambda}\left[N'N - \lambda E_{vv}\right]$$

$N'N=(r-\lambda)I_v+\lambda E_{vv}$ …(5.15)

From (5.14) and (5.15), we get for a symmetric BIBD

N'N=N'N

Thus, the inner product of any two rows of N is equal to the inner product of any two columns of N, i.e., λ.

Hence, in case of a symmetric BIBD, any two blocks have λ treatments in common.

Reference Books

Das, M.N. and Giri, N.C. (1988) Design and Analysis of Experiments(2nd Edition). New Age International, New Delhi.

Douglas,C. Montgomenry (2012). Design and Analysis of Experiments. John Wiley and sons, New York.

Gupta, S.C. and V.K. Kapoor(1999). Fundamentals of Applied Statistics (Third Edition), Sultan Chand and Sons, New Delhi.

Text Books:

1. Das M.N and Giri N.C (1986) Design and Analysis of Experiments, Wiley Eastern, New Delhi.
2. Gupta, S.P. and Kapoor, V.K. (1978): Fundamentals of Applied Statistics, Sultan Chand & Sons.
3. Panneerselvam, R. (2012): Design and Analysis of Experiments, PHI.

Reference Books:

1. Rangaswamy, R (2014). Text book of Agricultural Statistics, New Age publishers
2. Montgomery, D (1972) Design of Experiments, John Wiley and Sons.
3. Kempthorne, (1956) Design and Analysis of Experiments, John Wiley. New York.