MARUDHAR KESARI JAIN COLLEGE FOR WOMEN, VANIYAMBADI

DEPARTMENT OF STATISTICS

SUBJECT NAME : TESTING STATISTICAL HYPOTHESIS

SUBJECT CODE : CST61

CLASS : III B.Sc Statistics

SYLLABUS

UNIT V

Basic ideas on decision theory - Loss functions - Risk functions - Prior distributions - Bayes Risk - Simple problems based on Bayes estimation and testing.

Testing of Statistical Hypothesis

Decision Theory

Introduction :

Decision Theory (or the theory of choice) is a branch of applied probability theory and analytic philosophy concerned with the theory of making decisions based on assigning probabilities to various factors and assigning numerical consequences to the outcome.

Basic Elements :

There are 4 basic elements in decision theory

1) Acts

2) Events

3) Outcomes

4) Payoffs

What is Decision Theory ?

Decision theory refers to a range of econometric and statistical tool for analysing an individuals choices.

In other words, it lets the entity make the best logical decision possible when dealing with uncertain and unknown circumstances

Analysts worldwide use decision theory to understand better how people make decisions and the market trend to make better commercial judgements.

Two types:

1) Normative Theory
2) Optimal theory

Normative Decision Theory:

This decision analysis theory analyses the repurcussion of ideal logical decisions based on a set of values.

It also decides the optimum course of action based on limits and assumptions.

However, it does not consider other factors interfering with it. Instead, it deals with expected behaviour, decision-making process and the best potential.

This theory employs tools, procedures and computer applications to arrive at an optimal decision.

Optimal decision theory:

This quantitative and qualitative method, often known as descriptive decision theory, examines the decisions made by irrational people.

Furthermore, this theory employs various frameworks hypothesis and functions to comprehe[nd] practical actions that follow a set of norms.

In Bayesian statistics, Bayes rule prescribes how to update the prior with new information to obtain the posterior probability distribution, which is the conditional distribution of the uncertain quantity given new data. There are many ways to construct a prior distribution. In some cases, a prior may be determined from past information such as previous experiments. A prior can also be elicited from the purely subjective assessment of an experienced expert.

When no information is available an uniformative prior may be adopted as justified by the principle of indifference.

In modern applications, priors are also often chosen for their mechanical properties, such as regularization and feature selection.

The prior distribution of model parameters will often depend on parameters of their own.

uncertainty about these hyperparameters can in turn be expressed as hyperprior probability distributions.

For example, if one uses a beta distribution of model parameters will often depends on parameters of their own.

Applications of Decision Theory :

* Mathematics
* Economics
* Business
* Data and Social Sciences
* Biology
* Psycology
* philosophy
* politics

Loss and Risk Functions :

A loss function is a map $l : \Theta \times A \to R$

Interpret $l(\theta, a)$ as the cost, if we make decision a when true state of nature is θ.

The risk function of a decision rule $d : \mathcal{X} \to A$ is defined by

$$R(\theta, d) = E_\theta \, l[\theta, d(x)] \quad \theta \in \Theta$$

$E_\theta h(x)$ is the expectation of $h(x)$ when $x \sim f(x$

$R(\theta, d)$ = expected loss of rule d when applied to observation $x \sim f(x/\theta)$

Continuous case $R(\theta, d) = \int l[\theta, d(x)] \, f(x/\theta) \, dx$

Discrete case $R(\theta, d) = \sum_x l(\theta, d(x)) \, P(x/\theta)$

Prior Distribution :

A prior probability distribution of an uncerta quantity often simply called the prior is its assume probability distribution before some evidence is taken into accounts.

P is a parameter of the underlying system (Bernoulli distribution) and

α and β are parameters of the prior distribution (beta distribution) hence hyperparameters.

In principle, priors can be decomposed into many conditional levels of distributions, so called hierarchical priors.

Base Risk:

Risk-based decision making provides a process to ensure that optimal decisions, consistent with the goals and perceptions to those involved are reached.

This process ensures that all available information is considered and used as appropriate to the decision at hand.

Risk Assessment:

Risk assessment is the process of identifying potential hazards in the system and ranking them and in terms of risk characteristics as defined previously. As such it attempts to provide answers to the following questions.

* What can go wrong?

Risk Management:

Once a screened and prioritized list of risk has been developed, a risk management action plan can be developed. As the risk countermeasures will vary widely for different situations, no comprehensive list of potential management attempts to provide answers to the following questions

1) What can be done

2) What options are available and what are their associated tradeoffs?

3) What are the effects of current decisions on future options?

Problem:

It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam mails before they reach your inbox. A certain brand of software claims that it can be detect 99% of spam emails and the probability for a false positive (a non spam email detected as spam) is 5%.

Now, if an email is detected as spam, then what is the probability that it is in fact a non-spam email?

Solution:

Let A = event that an email is detected as spam

B = event that an email is spam

B^C = event that an email is not spam.

$P(B) = P(B^C) = 0.5 \quad P(A/B) = 0.99 \quad P(A/B^C) = 0.05$

Hence by. $P(B^C/A) = \dfrac{P(A/B^C)\,P(B^C)}{P(A/B)P(B) + P(A/B^C)P(B^C)}$

$= \dfrac{0.05 \times 0.05}{0.05 \times 0.05 + 0.09 \times 0.5} = 5/104$